# Modelling Human Fairness in Cooperative Games: A Goal Programming Approach

by

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# Abstract

The issues of rationality in human behavior and fairness in cooperation have gained interest in various economic studies. In many prescriptive models of games, rationality of human decision makers implicitly assumes exchangeability. This means that real people are assumed to adopt the beliefs of a player as expressed in the game when placed in the shoes of that particular player. However, it is a well debated topic in the literature that this modeling assumption is not in accordance to what behavioral economists have observed in some games played with real human subjects. Even when assuming the role of the same player in the game, different people think differently about the fairness of a particular outcome. People also view fairness as an essential ingredient of their decision making processes in games on cooperation.

The aim of this research is to develop a new modeling approach to decision making in games on cooperation in which fairness is an important consideration. The satisficing and egilitarian philosophies on which weighted and Chebyshev Goal Programming (GP) rely, seem to offer an adequate and natural way for modeling human decision processes in at least the single-shot games of coordination that are investigated in this work. The solutions returned by the proposed GP approach aim to strike the right balance on several dimensions of conflicting goals that are set by players themselves and that arise in the mental models these players have of other relevant players.

Fairness concerns are important in the well-known Ultimatum and Dictator games. These games are modeled using a Chebyshev GP approach. Parallels are drawn between the approach and concepts of human decision making from the field of cognitive neuroscience and psychology. The Chebyshev GP is the universal mechanism in the model for players to decide how fair outcomes are to be identified, but allows individuals to differ in their belief which outcomes are fair. Computer simulations of these GP models, testing a large number of Ultimatum, Dictator and Double Blind games, lead to distributions of proposals made and accepted that correspond reasonably well with experimental findings.

In our study of some simple but classic cooperative games, a fairness model is developed using weighted GP by taking into account players' aspiration goals and preferences in terms of profit and fairness concerns explicitly. The model offers a framework by which players can make decisions considering the different viewpoints of the potential partners of a coalition. The application of this framework to the Drug and Land games shows that the inclusion of fairness in the game produces solutions that may sometimes deviate significantly from solutions obtained from standard methods of cooperative game theory.

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# Declaration

'Whilst registered as a candidate for the above degree, I have not been registered for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award.'

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*To the memory of my mother,*

*Fatimah*

# 1

# Introduction

## 1.1 Overview

Characterising human behaviour in decision making is important in several fields of the applied sciences including economics, sociology, political science, operational research, and game theory. Mathematical and computer models have been developed in these fields in which the human aspect of decision making is placed at the centre of the problems investigated: agent-based technologies, models of game theory, and many models in economics and social sciences. Entire branches of theory are build on the concept that humans are (or aim to be) rational when making decisions. This is uncontroversial. The issue really is: how to capture rationality in a decision model?

Rationality often means that decision makers will want to take those actions that maximise their own expected payoffs (profit, utility) given their beliefs. From within a framework of assumptions about these players, their strategies, and their payoffs, the power of a decision model can be twofold. First, if the decision problem is complex, the mathematical model may have the ability to identify optimal outcomes or strategies for the players that they themselves would find difficult to identify. The model is then especially praiseworthy if its solutions are much better. Second, a model may be able to explain why certain outcomes are observed in reality, and why e.g. rational play does not always lead to desirable outcomes. We can say that the model has great *prescriptive power*.

Difficulties may arise, however, when the model predicts outcomes that are not being observed in reality. If this can be explained by the lack of humans to think through all the effects of their decisions and of relevant others to their logical end, then this is perhaps not such a bad thing if humans are then accepting the logic of the model and change their strategies accordingly. In fact, this approach is a cornerstone of scientific progress in many fields.

Behavioral economists have observed decision making not in accordance to the prescriptive solutions for some very simple cooperation games played by real people. It seems quite difficult to convince humans to change their strategies as to match the prescriptive optimal strategies. A logical conclusion is that the model should be altered. Perhaps the payoffs were not reflecting the true utility of the players? Perhaps a player in an economic game is not concerned about its own monetary profit, but rather wishes to see profits fairly distributed amongst players? Perhaps a player adopts this thinking by making assumptions about the impact that current decisions might have on the future, while the model itself forgot to include this? It might be that players let their decisions be influenced by unconscious processes in the brain. Looking back on the strategies they played, they might identify their play as ruled by emotions and hence as being based on 'irrational' or 'semi-rational' beliefs. It can still be that despite this awareness, they would be quite reluctant to rule out the importance of these emotions in driving their decisions. In this dissertation, we focus on human decision making in relatively simple problems of cooperation with other humans where these issues are at play.

## 1.2 Research Objectives

The issue of what rationality means and how its shapes decision making in games has been a hot topic of discussion in the literature. From studying the ultimatum game, it seems that fairness concerns play a role in decision making situations in which it 'seems' totally misplaced. The thesis aims to:

- Identify the reasons and build-up an understanding of why humans have this ‘build-in’ capacity or drive for fairness.

- To formulate novel decision models incorporating fairness measurements for each player in the ultimatum game and some of its variations.

- To compare the solution predicted by these models with real-life experimental data available from the literature.

## 1.3 Thesis Contribution

In the first part of the thesis, theories of human behaviour and decision making are reviewed. It reveals the belief that the essential framework of how humans make decisions is universally shared amongst all humans. This also strengthens the belief that the idea of fairness, and the mechanism people use in their mind of how to identify fair outcomes or strategies, is rather universal. The models in this thesis exhibit this feature.

It is also clear from the literature study that humans think differently about which outcomes they see as fair or unfair. This is not only shaped by the society they live in and by a universally shared difference in perception of the severity of being treated unfairly by someone else versus treating someone else unfairly, but by individual attitudes. The models in this thesis also exhibit this characteristic: the solutions returned by a universal fairness algorithm is highly dependent on the particular player that solves it. Thus, instead of postulating which outcomes are fair from the start and then study how humans seem to want to achieve these outcomes, it seems more appropriate to study how people actually make decisions that they think are fair, and then reflect on what the consequences are. Are these solutions fair from a societal point of view? Do players need to have the same perception about which outcomes are fair in order to reach cooperation and be successful as an individual?

In the search for descriptive game theory models that can explain experimental results in economic games, Goal Programming (GP) has not yet been explored. It is argued in this thesis that GP allows for a high-level description of key elements put forward in field of theoretical cognitive neuroscience, including the concepts of goals, efficient biological computation, theory of mind, and reward prediction error mechanisms. It is shown that the GP framework can implement, within the decision model of a player, mental models that this player has about the desires of other players. It is demonstrated how the GP framework can be used to construct mathematical approaches for implementing theories about fairness that have been developed in the fields of behavioural economics and evolutionary psychology. The GP framework allows for diversity in beliefs and values between players, and shows this to be essential for explaining experimental finding through a model.

The comparison with standard approaches in cooperative game theory helps to identify a roadmap for future research to incorporate fairness modelling in the standard theory. It seems important to model the different mindsets of individual players more accurately in order to explain real behaviour or provide prescriptive models for optimal strategies in particular contexts. At a more general level, such models may help to shed light on what attitudes are favorable in the process of negotiating cooperation deals, and in what kind of societies successful cooperation deals are more likely to occur.

## 1.4 Thesis Structure

The remainder of the thesis is organized as follows:

**Chapter 2** Relevant literature is reviewed. The chapter looks at the importance of 'fairness' concerns observed in experiments with real subjects, as well as how fairness has been modelled in economic games through the adoption of new utility functions. Scientific theories on human behaviour and decision making are reviewed, as well as some aspects from the fields of (evolutionary) psychology, theoretical cognitive neuroscience, and economics. Goal Programming is introduced.

**Chapter 3** Classical cooperative game theory is discussed, and applied to the examples of the Drug Game and Land Game. The limitations of classic solution concepts with respect to their ability to reflect fairness concerns is discussed.

**Chapter 4** A Goal programming (GP) framework on how humans make decisions is presented, drawing parallels with the field of cognitive neuroscience. GP seems able to capture, at a very abstract level, the concepts of multi-valued aspects of the goal state, efficient biological computation, and other concepts put forward in the field of neuroscience.

**Chapter 5** The GP framework of Chapter 4 is applied to the Ultimatum Game and some of its variations. The different goals are specified based on the drivers identified in the field of evolutionary psychology. The numerical results are distributions of outcomes rather than a single outcome. These distributions are compared with experimental findings with human subjects reported in the literature. A sensitivity analysis is conducted to get insights on the consequences of fairness beliefs for individual and societal welfare.

**Chapter 6** The fairness GP model with pooling formulation is introduced to explain human cooperation in the Drug Game and Land Game. Through numerical examples is illustrated of the importance of players' sharing compatible beliefs about fairness with respect to the formation of a pool and its payoff distribution.

**Chapter 7** Conclusions and directions for further research are presented.

# 2

# Literature Review

## 2.1 Introduction

This chapter reviews relevant literature. The main research question to be addressed is getting an insight into what drives humans to consider fairness in their decision making. As Goal Programming is the main methodology for modelling used in this work, it is also reviewed in this chapter.

## 2.2 The Ultimatum Game

Game theory deals with human decision making in situations whereby the intentions of other players need to be considered. The Ultimatum Game (UG), Dictator Game (DG) and Double-Blind Dictator Game (DBDG) are simple but yet powerful games that have received much attention among researchers. They can be defined as follows:

- UG. One participant gets a (large) sum of money to divide between himself as a proposer and another participant as a respondent. If the respondent accepts the offer, both participants gain the amount agreed upon. If the responder rejects the offer, neither participants gets anything (Guth et al., 1982).
- DG. A variant of the UG in which the proposer can simply divide the sum between the two players and there is nothing the respondent can do about it.
- DBDG. A variant of the DG in which proposals from many players are sealed and neither the respondent nor the experimenter knows which proposer offered how much.

A considerable number of studies have been conducted to study fairness issues using the UG (Camerer, 2003, Roth et al., 1991). If players are pure profit maximisers, and they know this to be true of each other, then the UG, DG, and DBDG would all direct to the same behavior: proposers keep most of the money, and responders would accept the bit that is left, no matter how small. This does not correspond to what is observed. In the UG, proposers tend to offer up to half of the total sum (Polezzi et al., 2008), and respondents typically do not settle for much less than half. These contradictions have been widely discussed in the literature (Camerer, 2003, Colman, 2003).

The early study by Guth et al. (1982), which is remarkably robust considering the amount of UG studies that followed (Haselhuhn and Mellers, 2005), demonstrates intriguing results. It reveals that the proposers offer far more than what responders would still accept. In fact, proposers offer nearly half, while the responders are not likely to accept offers less than twenty percent. Levels of offers and rejection rates vary across cultures (Henrich et al., 2005). The modal offer made by adult proposers in industrial societies is nearly half, whereas the mean offer is approximately 60% (Camerer, 2003, Roth et al., 1991). Roughly half of the responders reject unfair offers below 20% (Roth et al., 1991, Yamagishi et al., 2009). In the DG, proposers offer much smaller amounts, but typically the offers are more than they have to be. In the DBDG, proposers do keep most of the money for themselves.

Decision makers in these games therefore must have other considerations they take into account (Camerer and Thaler, 1995). There is a considerable body of literature on the features that are thought to be of importance. One obvious strategy in the search for descriptive game theory models that can explain experimental results, is to deal with players' utility functions where the utility of one player depends on the payoff of other players. In particular, fairness intentions have been incorporated such that players derive utility from punishing other unfair players or from rewarding other fair players, even when this affects their own monetary payoff (Rabin, 1993), and similarly, utility can also be derived from the adherence to social norms (Fehr et al., 2004).

## 2.3 Incorporating Fairness in Economic Games

The importance of fairness, and the pressure it brings onto the fairness of solutions or outcomes, has been discussed in relation to many specific real world problems. Fairness has been raised, for example, in the context of queueing systems (Raz et al., 2004), scheduling (Baruah et al., 1997), bandwidth allocation (Ogryczak et al., 2008), and congestion control (T. and R., 2004). In general, win-win solutions are thought to be fair solutions in these cases, not just because everyone gets what he or she wants but in addition none of the members are exploited (Muller et al., 2008).

In social economic research, fairness is translated into two commonly applied approaches known as distribution models and reciprocal kindness models, respectively. Distribution models (Bolton, 1991, Bolton and Ockenfels, 2000, Fehr and Schmidt, 1999) imply that people care about their payoff compared to others. Reciprocal kindness models (Rabin, 1993) demonstrate the importance of how each player views the intention of the other to demonstrate a fairness equilibrium. Both kinds of models have been constructed to help explain the 'anomalies' in the ultimatum and dictator game when played by real subjects.

Perhaps the first model that includes the idea of reciprocal kindness is Rabin (1993). Instead of being pure individual profit maximisers, the model assumes that people can also be motivated by fairness considerations. The model requires an explicit representation of fair intentions and is applicable only for zero-sum games between two agents.

In distribution models, it is assumed that people strive for egalitarian outcomes. In Bolton (1991)'s model, the assumption is that if a subject get less than half, then the utility of receiving the money is decreasing the larger the difference relative to what the other party receives. This implies envy. If the player gets more than half, her utility can only be increased if she could earn more. These assumptions model an asymmetric attitude towards fairness. Relative comparison matters a lot when one feels unfairly treated, but matters very little when one feels fairly treated (Camerer and Thaler, 1995).

Bolton and Ockenfels (2005) captures both features of the fairness measurement by developing two simple archetype models, one distribution-based and one kindness-based. In their first model, fairness is measured in terms of the relative payoff comparison. In the second, the measure of kindness is invariant with respect of characteristics on the strategy space such as volition.

Falk et al. (2008) claims that the inequity aversion models of Bolton and Ockenfels (2005) and Fehr and Schmidt (1999) are incomplete as they neglect fairness intentions. It is also claimed that the models by Rabin (1993) are incomplete because the models are exclusively based on intention-driven reciprocal behavior. The result from the experiments, it is argued, shows that subjects exhibit weakly reciprocal behavior even if they cannot attribute fairness intentions. They claim the models from Falk and Fischbacher (2006) captures both of these aspects. The models in particular explain why there is little cooperation in the absence of a punishment opportunity, but a lot when punishment is possible. Note that not cooperating with a coalition (or a veto), can be seen as a kind of the punishment that one player can impose upon the other members of a coalition.

Clearly, a challenge to model fairness in relation to economic decision making is the lack of an agreed upon formal measure of fairness. Several solutions have been proposed to incorporate the findings of the Ultimatum Game and its variations into decision models.

### 2.3.1 Inequity Aversion Model

Fehr and Schmidt (1999) modify the utility function of players by allowing it to depend on the payoffs of other players. In the UG, each player's utility function now depends on what both players receive. Fehr and Schmidt (1999) propose the "inequity aversion" model in which a player has a disutility of receiving a pay-off that is different from the other players. The extent of disutility depends on the player's relative payoff position. Players exhibit a stronger disutility from having received less than the others than having received more.

Fehr and Schmidt (1999) have used a series of economic-exchange games to probe the human instinct to be fair and to punish those who are not. The Fehr and Schmidt (1999) inequity aversion model allows asymmetry in fairness perception by different types of players. The theory assumes that humans are not purely rational and self-interested, and that they base their decisions in part on making sure that they receive a fair share. Decision makers prefer to some extend to minimize the difference between their own monetary payoff and others. They are willing to give up some material payoff to move in the direction of more equitable outcomes. In this model, player $j$ has a preference or utility function of the form:

$$U_j(\pi) = \pi_j - \alpha_j[\frac{1}{n-1}] \sum_{k \neq j} max[\pi_k - \pi_j, 0] - \beta_j[\frac{1}{n-1}] \sum_{k \neq j} max[\pi_j - \pi_k, 0]$$

where

$\pi_j$ - payoff of player j

$\pi_k$ - payoff of player k

$\alpha_j$ - parameter measuring how much player j dislikes having less money than others.

$\beta_j$ - parameter measuring how much player j dislikes having more money than others.

$\beta_j \leq [\alpha_j, 1]$

$n$ - number of players.

The second term in this equation measures the utility loss from disadvantageous inequality, while the third term measures the loss from advantageous inequality. For the case of $n > 2$, player $j$ compares his income with all other *n-1* players. In this case the disutility from inequality has been normalized by dividing the second and third term by *n-1*. This is necessary to make sure that the relative impact of inequity aversion on player $j$'s total payoff is independent of the number of players. Furthermore for simplicity, the disutility from inequality is self-centered in the sense that player $j$ compares himself with each of the other players, but he does not care per se about inequalities within the group of his opponents.

There are two assumptions in this model: First, $\beta_j < 1$, which shows that players are not willing to sacrifice all their own payoff for eliminating advantageous inequity. Second, $\alpha_j \geq \beta_j$, which means that players care more about disadvantageous inequity than advantageous inequality.

Charness and Rabin (2002) extend the inequity aversion model by incorporating reciprocity in the utility function. This generalised function allows players to reciprocate when others have been nice or mean towards them.

### 2.3.2 Equity-Reciprocity-Competition Model

Bolton and Ockenfels (2000) propose the Equity-Reciprocity-Competition (ERC) model in which each agent's utility function depends on her absolute payoff as well as her relative share of the total payoff. Under ERC, given an absolute payoff, an agent's utility function is maximised when her share is equal to the average share.

### 2.3.3 Distributional and Peer-Induced Fairness Model

In both the inequity aversion and ERC models, Ho and Su (2009) argue that agents' social preferences depend on payoffs of other economic agents, and call these *distributional* fairness concerns. This study argues that in many situations, people are also driven by social comparison. That is, they have a drive to look to others who are in similar circumstances (i.e. their peers) to evaluate their outcomes and judge whether they have been treated fairly. They thus develop a model to include both distributional and *peer-induced* fairness concerns. The model uses the signal concept between players in games in an aim to better understand when offers are accepted or rejected.

### 2.3.4 Pleasure Model

Haselhuhn and Mellers (2005) approach economic games such as the UG and DG from the viewpoint that emotions play a central role in decision making. They distinguish between strategic and non-strategic pleasure. Strategic pleasure is the expected pleasure of offers, whereas non-strategic pleasure is the pleasure of accepted payoffs. They suggest that pleasure of a pay-off is a tradeoff between fairness and selfishness as follows:

$$P_{ij} = w_{Fi}F_j + (1 - w_{Fi})S_j \tag{2.1}$$

where $P_{ij}$ is the rank order of person $i$'s pleasure with payoff $j$, $w_{Fi}$ is a relative weight of fairness, $F_j$ is the rank order of payoff $j$ based on pure fairness, and $S_j$ is the rank order of payoff $j$ based on pure selfishness.

Like pleasure judgements, preferences or choices over offers are tradeoffs between strategic pleasure (the expected pleasure of offers) and non-strategic pleasure (the pleasure of accepted offers). This trade-off is expressed as:

$$C_{ij} = w_{Pi}P_{ij} + (1 - w_{Pi})EP_{ij} \tag{2.2}$$

where $C_{ij}$ is the choice order of offer $j$ for proposer $i$, $w_{Pi}$ is the relative weight of pleasure (for proposer $i$), $P_{ij}$ is the rank order of pleasure for payoff $j$, and $EP_{ij}$ is the rank order of expected pleasure for offer $j$, and is:

$$EP_{ij} = s_{ij}P_{ij} + (1 - s_{ij})PR_i \tag{2.3}$$

where $s_{ij}$ is proposer $i$'s subjective belief (measured as a probability from 0 to 1) that a responder will accept offer $j$ and $PR_i$ is displeasure if proposer $i$'s offer is rejected. The subjective belief that a receiver will accept any offer is 1.0, so choices would be a direct function of the pleasure of accepted offers (i.e. $C_{ij} = P_{ij}$ in equation 2.2, and $EP_{ij} = P_{ij}$ in equation 2.3). The pleasure in this context refers to feelings that people experience after having achieved their targets. They based this assumption on the observations that fairness in solutions led to higher a posteriori happiness ratings. In this research, fairness is said to be achieved when everybody achieves their target and their preferences have been met.

According to Ostmann and Meinhardt (2007), if one is concerned about fairness standards, then in order to judge the fairness of a proposal, one needs information on one's own payoff share and the payoff share of the opponents, as well as a set of subjective or objective principles to specify rules of arbitration. Bereby-Meyer and Niederle (2005) also state that instead of thinking about their own payoff, people also compares to other people's payoff. The perception of being treated unfairly can cause conflict, and consequently interfere with efficient fulfillment of the cooperative motive.

Some studies suggest to go beyond an inequity aversion approach. De Jong et al. (2008), for example, claim that inequity aversion might not be sufficient to describe fair deals and calls for the inclusion of additional information. They introduce the priority awareness model to model people's perception of fairness in the presence of additional information using the notion of priorities.

In the context of public good games, Tabibnia and Lieberman (2007) define fairness as the equitable distribution of goods or outcomes, and define cooperation as doing one's share to maximize public goods rather than working individually to maximize personal goods. They reviewed studies that employed a social cognitive neuroscience method to investigate the effective impact of fairness and cooperation in collaborative settings. These studies reveal that fairness and cooperation activate the same hedonic regions of the brain as financial gain, and indicate that these factors may merit equal consideration. Although evidence suggests that receiving an unfair proposal may be related to negative emotional responses, until recently it was unclear whether fair offers produced positive emotional responses beyond those associated with the monetary payoff that is associated with fair offers.

## 2.4 Experimental Study in Economic Games

The literature of experimental gaming testifies to the fruitfulness of empirical research for developing a game-theoretic framework. Experimental evidence can contradict conventional theory. The question arises whether the experiments are badly executed, or the theories are based on the wrong assumptions, or both? Assuming that the experiments hold the truth, recent research has aimed to overcome the discrepancies between standard game theoretical predictions and experimental observations by altering the underlying utility function of the subjects in the models.

There is experimental evidence that symmetry of players does not always hold. This leads to models with heterogeneous preferences to capture individual desires and behavior (Dannenberg et al., 2007).

There is also evidence from the experiments that emotions somehow play a role. Behavioral game theory therefore expands analytical theory by adding emotions, or how players feel about the payoffs other players receive (in relation to their own payoffs), as well as constructing more accurately the ways in which people deploy (limited) strategic thinking and learning from experience (Camerer, 2003). This view is also supported by Sanfey (2007) who studied the human brain activity while playing ultimatum game. It showed that the brain areas related to both cognition and emotion are strongly involved.

In standard economic analysis, the rational dictator in the Dictator Game should take all the money for himself, leaving nothing for the recipient. In fact this finding contrasts laboratory studies (Forsythe et al., 1994). The results show a wide dispersion of dictator game giving in the range of zero to fifty percent (Bolton et al., 1998).

## 2.5 Computational Theory of Mind

Can computer models of the decision making process ever be as good as human decision making? To many this may not seem to be a scientific question. In particular, influential thinkers throughout the centuries have clearly argued that the human mind (and thinking) is separate from the physical world (Pinker, 2002). Since software programs run on computers, clearly physical objects, they will thus never be able to replicate the human mind.

It may be true that computers will never achieve this task. However, recent frontiers of knowledge - the sciences of mind and brain - are indicating that it is at least a valid scientific quest. This multi-disciplinary field (comprising of cognitive science, cognitive neuroscience, and psychology) provides more and more facts about how the mind operates. According to Pinker (2002), five ideas from this field have revamped about how we think and talk about minds:

- Idea 1. The mental world can be grounded in the physical brain by the concepts of information, computation, and feedback.
- Idea 2. The mind is not a blank slate, because blank slates don't do anything.
- Idea 3. An infinite range of behavior can be generated by finite combinatorial programs in the mind.
- Idea 4. Universal mental mechanisms can underlie superficial variation across cultures.
- Idea 5. The mind is a complex system composed of many interacting parts.

According to cognitive science, the mind can thus be compared a something like a computer in the sense that thinking is processing of information using processes that can be described by a set of algorithms. It thus theoretically possible to construct computer models of the mind and of human decision making processes.

Sally and Hill (2006) conducted one study that examined how the cognitive ability to infer the mental states of others affects fainess-related behaviour among children with and without autistic spectrum disorder (ASD). Not all scientific thinkers of today adhere to this computational theory of mind; see e.g. Penrose (1989). It is not the purpose of this research to contribute to this debate on the philosophical level, but to use the ideas of the computational theory of mind as a framework to construct better decision support models, in particular models about cooperation.

Scientists in the field of theoretical neuroscience strongly adopt the view that the mind is equivalent in much of its functions to an algorithmic process which leads to a computational theory of the mind, suggesting that computer models may be built mimicking our thought processes, see Montague (2007). So what would be essential ingredients for such a computer program? This requires some deeper understanding of the theoretical findings in this field of neuroscience.

Theoretical neuroscience is developing a theory of how the human brain makes decisions. A key concept in this theory is *efficient biological computation*. A human brain is a biological computer in the sense that when a human brain is thinking it performs calculations. In contrast to silicon computations, however, biological computations are not lifeless streams of symbols, totally devoid of meaning; nature has equipped biological computations with a measure of their value - an extra measure of their overall worth; a measure of the value of that computation to the overall success of the organism, its overall fitness. *Efficiency in biological computation aims for the best long-term returns from the least immediate investment.*

Preservation of energy is thus a principle that not only guides our body (e.g. our heart rate only goes up when in situations where this is needed) but also our brain and mind. Biological organisms compute slowly and gently in order to preserve energy as it may not be sure when the next recharge will happen, but only as slowly and gently as is consistent with its own survival.

Efficiency also explains why we use noisy and ambiguous communication using as few words and repetition as possible to get the message across. The reasons why the recipient often gets the meaning is because as communicator we build a mental model of the recipient so we can assess what should be sufficiently clear, and likewise the recipient builds a mental model of the communicator which will help in the understanding of the message. These mental models of other people or situations are only as detailed as deemed necessary, usually only 'sketches' where details are only 'filled in' (computed) if considered important.

Essential to efficient (biological) computation is having goals (Montague, 2007); without goals, a computational system cannot be efficient for the simple reason that it has nothing against which to gauge its ongoing performance. Goals express that we want to do some things more than other things. The nervous system implements goals by using collections of corrective guidance signals (error signals) to navigate an individual's behavior. The error signals tell the system to adjust when deviations from the goal state occur. The deviations from the goal state provide a model for the concept of desires, which inform the system how to adjust in order to move closer to achieving the goal state. Goals that people have change over time. As time goes by, the brain actually learns a vast amount of new values - it learns what it should 'want'.

Based on Montague (2007), we have extracted the following neuroscience principles about the human decision making process:

- Principle 1. At any time, humans have a set of (long or short term) goals or values in mind and these goals may change over time.
- Principle 2. Humans build mental models of experienced, actual, or imagined situations, actions they can take, and thought processes of other humans.
- Principle 3. Always and automatically, some level of desirability (a valuation) gets associated with each mental situation and action.
- Principle 4. Desirability is multi-valued and depends on which values (goals, decision criteria) are currently playing a major role in the mind.
- Principle 5. Decisions are made based on a reward prediction error mechanism: we are urged into action according to the magnitude of an error signal which measures the distance between the current situations

from the target situation (the goal state).

The thinking processes involved in all of the above are governed by the principle of efficient biological computation - the best long-term returns from the least immediate investment. This may explain why sometimes we take a certain action based on actually only vague mental models or without having considered all alternative actions in detail and still feel very confident about the decision taken.

All sorts of things can make this mechanism not working properly. Depression, for example, is thought be associated with error signals which are too low so that the individual is not urged into any form of action (Montague, 2007). Even individuals of normal health make mental models about desired futures which are usually far off from how we will really think once we get to that time in the future, and it is one of the reasons why we seem constantly in pursuit of happiness (Gilbert, 2007).

Brains with the right combination of cognitive and emotional faculties benefit from cooperation. People display a sense of morality, justice, and community, an ability to anticipate consequences when choosing actions, and a love for children, spouses, and friends. We engage in violence or in works of peace depending on which set of motives is engaged in the brain. Psychology does not rule out our "free will" or responsibility; it helps to provide an explanation of certain behavior, it does not exonerate the behavior. Culture can be seen as a pool of technological and social innovations that people accumulate to help them live their lives. Laws translate our sense of justice into fine-tuned deterrence policies for those people who could have been deterred by it.

Our desire for living in peace, and cooperation, has therefore ultimately its explanation through the existence of certain intuitive reasoning faculties and emotions of the mind.

Psychology has produced a tentative list of core cognitive intuitions that was suitable for the world in which our ancestors lived: an intuitive physics, an intuitive version of biology and natural history, and intuitive engineering, and intuitive psychology, a spatial sense, a number sense, a sense of probability, an intuitive economics, a mental database and logic, and language. To narrow the list down to perhaps the three most important intuitive cognitive faculties with respect to cooperation, we would have (descriptions from (Pinker, 2002)):

- An intuitive psychology, which we use to understand other people. Its core intuition is that other people are not objects or machines but are animated by the invisible entity we call the mind or the soul. Minds contain beliefs and desires and are the ultimate cause of behavior.

- An intuitive economics, which we use to exchange goods and favors. It is based on the concept of reciprocal exchange, in which one party confers a benefit on another and is entitled to an equivalent benefit in return.

- A mental database and logic, which we use to represent ideas and to infer new ideas from old ones. It is based on assertions about what's what, what's where, or who did what to whom, when, where, and why. The assertions are linked in a mind-wide web and can be recombined with logical and causal operators such as AND, OR, NOT, ALL, SOME, NECESSARY, POSSIBLE, and CAUSE.

Our intuitive cognitive skills are often insufficient to grasp many domains of knowledge, including modern physics, economics, mathematics, and human understanding itself. Science and education is needed to try to make up for what the human mind is innately bad at. Understanding the difference between our intuitive ways of thinking and the best science can help us make better decisions.

### 2.5.1 Mentalising

According to Frith and Frith (2006), most studies so far have made little attempt to isolate the aspects of mental perspective. In Frith and Frith

(2003), mentalising refers to the process of thinking about another person's mental perspective and thereby predicting what they (can) know or (may) want. Mentalising applies to any agent that is observing, reading about, or interacting with a member of his or her 'own' group or an out-group.

The bottom line difference that the idea of mentalising brings is a prediction of what other individuals will do in a given situation based on a prediction of their desires, knowledge and belief, rather than from an actual (or potential) state of the world (Frith and Frith, 2006). The ability to think about other people might be an aspect of the ability to re-describe events from several points of view. Such ability that might fuel the emergence of science, art and culture in general.

Our ability to read the other player's thinking and intentions affects negotiations. It influences cooperation in that players' perceptions on fairness can be explicitly considered. This ability has been discussed widely in neuroimaging studies by numerous researchers, see e.g. Camerer et al. (2005), Lee (2005), Polezzi et al. (2008), Sanfey (2007), Sanfey et al. (2006, 2003). Studies reveal insights about the neural processes underlying our ability to predict 'what happens next' in a social interactions. Social interaction involves predicting a person's movements, intentions, bodily states and mental states. This ability of thinking about the content of other specific people's mind is in general referred to as 'theory of mind' or 'mentalising'. The recent research interest in economic study has focused on the neural correlates of the underlying decision making processes. The study from Sanfey (2007) reveals that the negative emotional state for automatic responses of decision making in responders' brains is consistently higher in the ultimatum game if they receive unfair offers.

### 2.5.2 Rationality and Psychology

Rational behaviour is assumed to be that decision makers act in a desire to maximise their expected utility given their beliefs. With a game theory context, players also consider the intentions of the other players, and the interrelationships these bring to the decision making problem. Arguably, rational decision making involves consistently making the right choices in the pursuit of desires. Rationality is perfect when decision makers are capable of making complex evaluations of all possible outcomes and choosing the best strategy to play.

It is recognized that people are not always able to make perfectly rational choices. Simon (1955) introduced the term *bounded rationality* to describe that people face limitations in their ability to gather and process information, and formulate and solve decision models. As a result, rather than to identify an optimal solution, people often adopt a strategy which attempts to meet certain criteria for adequacy, a strategy Simon called 'satisficing' as a portmonteau of 'satisfy' and 'suffice'. For example, satisficing is said to occur when people sell their house, when companies sell at a set price, or in consensus building when a coalition looks towards a solution everyone can agree on even if it may not be the best.

Some authors (Byron, 1998) see satisficing as part of an overall rational scheme in which people do optimize with respect to a more global utility function which incorporates the balance between the cost-saving aspect of satisficing and the risk of not optimizing. In this view, *satisficers are rational optimizers with respect to their own global utility function.* Per definition, this function is likely different from others people's global utility function. Furthermore, this function can not be but a rough approximation of the function from which the decision maker could derive his or her true optimum in the long run. The latter assumption has to follow directly from Herbert Simon's premise that we have a limited ability to gather and process all relevant information.

Not being rational can also arise from other aspects than merely our limited ability of information gathering and processing. *Cognitive bias* has been studied in the fields of cognitive science and social psychology, and covers a range of observed effects whereby people tend to base their judgments on biased goals or evaluation methods, including anchoring, confirmation bias, egocentric bias, and the wrong application of intuitive logic or probability.

There is yet another important element of human behavior that seems to interfere with being rational: *emotions*. Most people are emotional in many of their actions Simon (1955). Liking, anger, gratitude, sympathy, guilt, and shame are emotions known to influence our decisions. Altruism, defined as behavior that benefits others at a cost to the behavior, is equally important. Are emotions really irrational aspects of human behavior or do they have some root of rationality? Should emotions and cognitive bias, just like bounded rationality and satisficing behavior, be taken into account when building decision models, in particular models to be used in a business environment?

### 2.5.3 Mutualism and Altruism

Humans may evolve a willingness to do good deeds. Two basic forms can be distinguished (Pinker, 2002):

- Mutualism or symbioses is helping others while pursuing your own interests;
- Altruism is benefiting someone else at your own cost.

Mutualism arises among people who have common interests and their realization that cooperation can enhance their chances of success and where the additional cost from cooperating is negligible compared to its benefit. Typical examples include friends that share common tastes, hobbies, or enemies. Altruism can evolve in different shapes:

- Nepotistic altruism (Pinker, 2002) is based on kinship and is making sacrifices to help relatives and family;
- Reciprocal altruism (Pinker, 2002) arises between parties who are not relatives but interact repeatedly;
- Reputation based altruism (Gintis, 2000, Nowak et al., 1995) is performed by a party who does a good deed for someone at a cost to themselves to signal other potential parties that they are good to cooperate with;
- Punishment based altruism ((Fehr and Gachter, 2002)) is performed by a party at a cost to themselves and is meant to punish someone who did not adhere to an expected form of cooperation.

Nepotistic altruism is very strong in both in animals and humans. Psychology explains this in terms of family relatedness; family members helping each other when the 'cost' to the helper is less than the benefit to the recipient discounted by their degree of relatedness (Pinker, 2002).

Reciprocal altruism, although present in some animals (bees for example), is far more present in humans, who relied on many different ways of equality matching transactions in ancient times, as it requires a brain with the right cognitive and emotional faculties. It arises when someone can give someone else a large benefit at a small cost to themselves. It needs a memory to keep track who did favors to whom and who does not return favors (cheaters). Those who reciprocate will favor from the exchange of good deeds and cheaters will not as they will be recognized and shunned or punished by the others. Mathematical models have indicated that cheaters will not necessarily completely disappear from the population but can remain in small numbers.

Reciprocal altruism can explain why we have several social and moral emotions (Pinker, 2002):

- Sympathy and trust: to prompt people to extend the first favor;
- Gratitude and loyalty: to prompt people to return favors;
- Guilt and shame: as a deterrence from hurting or failing to repay favors;
- Anger and contempt: to prompt people to avoid or punish cheaters.

Language helps us as well in doing good deeds at a small cost by giving others information, and it also helps to spread the word about who can be trusted (reputation) and who are cheaters.

Punishment may be performed by someone to signal others that you can't be messed about with - as a way to keep your reputation. Altruistic punishment, however, cannot be explained by signaling theory and may have to do more with enforcing some established social norms (Fehr and Fischbacher, 2004) about the type of behavior that is expected under cooperation.

The reliance of reciprocal altruism (and thus equality matching) on our emotions makes it vulnerable to conflict as people may feel differently about some situation. One may accuse someone of being non-friendly or non-loyal who thinks they are not; one can try to display sham emotions of guilt and shame to try to convince the other party that you will return favors even if you intend to cheat. Such behavior is intentional.

There is also a similar behavior that all people apply but subconsciously: self-deception. The function of self-deception is to maintain a positive self-image. In social psychology experiments (Pinker, 2002), people consistently overrate their own skill, honesty, generosity, and autonomy. They overestimate their contribution to a joint effort, explain their successes as a result of their own skill and contribute their failures to luck, and often feel that the other side has gotten a better deal in a compromise. Each party in a dispute can sincerely believe that the logic and evidence are on their side and that the opponents are deluded or dishonest, or both. Cognitive dissonance reduction is the process in which people change to whatever opinion it takes to maintain a positive self-image.

### 2.5.4 Long or Short-Term Goals

Cognitive neuroscience offers some insight as to why we sometimes take decisions which are not in our best interest in the long term but rather seem to be best only in the short term. Within the human brain, for example, the prefrontal cortex is believed to be largely responsible for our ability to reason and for considering the past and the future when making decisions. When this prefrontal cortex is working less, as in infants or when adults for example play computer games or take certain drugs, decisions will be taken based on instant satisfaction rather than long term goals, see Greenfield (2008), Chapter 13. Inside the human brain, several goals and desires struggle to get the upper hand (Idea 5), and as a result of neural processes some goals are becoming more important and playing the major role in the decision taken (Principle 1).

Such insights are often at the level of physical, chemical, and neural network processes, and while such findings are important to support the computational theory of mind, they are not completely satisfactory to explain why we wish to take decisions to satisfy short term goals rather than try to do best for the long term. More insight in this and other questions about emotions and altruism is offered by the field of psychology.

### 2.5.5 Social Behaviour

Modern psychology complements cognitive neuroscience and is compatible with the computational theory of mind, but offers an explanation in terms of beliefs and desires. It can explain that although people judgments' can be far from the truth, far from rational, be influenced by short term goals rather than long term goals, emotions and altruism, their decisions and actions may not be necessarily irrational or illogical.

Human behavior in psychology is explained by placing it in a social context of living in groups in which these human characteristics such as emotions and altruism helps us to live our lives. Since our current living patterns are in some respects much different from the past, the way we feel emotionally about certain things may not be fully in line with the current social and economic environment (Bechara et al., 2000, Loewenstein et al., 2001). The appreciation of fruits and fatty foods helped our ancestors to survive and replicate, but today it is being exploited by the food industry and rather a liability in our fight against obesity. Intuitive notions on probability were sufficient to help our ancestors in their daily lives, but are not up to the tasks university students currently need to undertake to understand modern theory of probability.

Reciprocal altruism (i.e. altruism towards others not related by kinship) is characteristic for humans and not present in most animals as it requires a brain that can implement a tit-for-tat strategy and can remember who returned favors and who did not (Fehr et al., 2004). Humans adopting reciprocal altruism can do much better in a society than humans who do not adhere to this strategy (cheaters).

Reciprocal altruism is driven by certain human emotions. Most people do have the *genuine* desire at least at some times to do good for others. Emotions and altruism are also genuine in the sense these are not under our control - we have them and they are difficult to hide. Real emotions are under control of the subconscious (such as the functions that regulate for example the heart rate) as opposed to sham (faked) emotions which are constructed by intention.

If humans are not selfish in nature, does this mean that their minds are build for doing good to the group (society) in which they life? This is a misunderstanding. Moral emotions, altruism, and ultimately the mind benefit the long-term interests of the individuals and not necessarily always the society in which they live. How the mind works and how it would be nice for the mind to work gives different answers.

In conclusion, psychology says that the brain is designed for fitness in the context of living in a group and in this sense our mind is remarkable and often seems to act rational. This does not mean that our brain is naturally build for finding the truth; indeed this provides the main reason for conducting scientific research. Building decision models based on how the mind works may therefore not necessarily be rational in the sense that it would be better in finding the truth than a model that is not aiming to mimic the human brain's way of thinking. Progress in science indeed often needs to go against our intuitive theories (on probability for example).

### 2.5.6 Human Cooperation

Cooperation has to do with transaction of goods (or services) between humans. Economists distinguish several types of goods:

- rival goods, made of matter and energy and if one person uses them another can't;
- non-rival goods which can be enjoyed by many at the same time and include information and ideas that can be duplicated at negligible cost; and
- public goods such as fresh air which can be enjoyed by a group but usually needs a form of care and then the question is whether all people who enjoy the good are willing to do an effort (those who don't are called free-riders who shirk).

The importance of acknowledging preferences of other players in the group can be illustrated by the well known "Orange Example" by Kersten in Vetschera (2009): two cooks negotiate how to share the last orange that is left in the kitchen, and finally they split the fruit in half. One cook then proceeds to put the peel of his half into a cake and throw away the inner part, while the other squeezes her half, uses the juice for a sauce and throws away the peel. Knowing the preferences of each other would have led to a superior division.

Studies by Dannenberg et al. (2007) identify the main influencing factors for the behavior of the subjects are the aversion someone feels when thinking about an advantageous inequity for the other players, and knowledge about the type of players they face. The results from this study show that, when players are informed about the type of the other players (cooperative, competitive, individualists), 'fair' groups (players are all cooperative) are more likely to cooperate in the final period of a public good game than groups of 'egoistic' players (all players are individualists or competive) and mixed groups. In absence of knowledge of the type of the other players, players tend to act more as individualists or competitive. Explicit information about the nature of your co-players thus has a significant influence on contributions made in these public goods games.

Recent economic research has thus stressed the importance of assumptions about the type of player and their typical associated intentions for judging co-players' actions and determining your own. This overshadows the standard economic approach judged by utility. This mental modeling of other players and their behavior plays a critical role in cooperation. The promotion and maintenance of a social relationship is important. The inequity-averse model by Fehr and Schmidt (1999), for example, is able to explain an impressive amount of experimental evidence not in line with the standard model of pure selfish behavior.

If all people were alike, it would be difficult to explain why we observe that people sometimes resist unfair outcomes or manage to cooperate even though it is a dominant strategy for a selfish person not to do so, while fairness concerns or the desire to cooperate do not seem to have much of an effect in other environments. The fairness consideration is at some points enforced by retaliation mechanisms. This phenomenon either can enforce desirable solutions or may have immediate negative effects.

The study of social dilemma games offer some insights into the human tendency to cooperate when there is a conflict between personal benefit and group benefit. Three types of social orientation have been identified:

- Cooperative individuals, who are concerned with maximising the outcomes for both themselves and others.

- Competitive individuals, who maximise the difference between outcomes for themselves versus others.
- Individualists, who maximise their own outcome with no regard for the outcome of others.

The differences between the three types of orientation determine the way people transform and respond to interdependent situations in social dilemmas. Considering these differences in social orientation in models may give us one way to look at and implement fairness in cooperative games on public goods. A question of interest is whether cooperation is more successful between individuals with the same social orientation than between individuals with different social orientations.

According to social dilemma theorists, one of the most promising solutions is to strengthen the group ties and increase people's identification with the group, so that members become motivated to contribute to the group. De-Cremer and van Vugt (1999) investigate why social identification might promote voluntary cooperation in social dilemmas. Researchers from the area of psychological studies have shown that members who strongly identify with their group may reduce the psychological distances between the group members so that they perceive each other as similar in terms of their goals and achievements.

## 2.6 Goal Programming as a Decision Making Tool

Since it is often difficult to find a reasonable solution by using traditional single-objective mathematical programming techniques, multi-objective programming is now widely recognized to be effective in solving real-world problems. Multi-objective optimisation belongs to the field of multi criteria decision aid. In multiple criteria optimization, optimal decisions have to be found in the presence of several conflicting criteria. A decision is only considered optimal if an improvement of one criterion implies a deterioration of at least one other criterion. The corresponding outcomes are Pareto efficient or Pareto optimal.

GP is a practical appproach towards multi-objective optimisation. It is an approach to allow decision makers to find satisficing solutions according to the philosphy of Simon (1955). Rather than searching for the complete set of Pareto efficient solutions, it incorporates information on the preference structure of decision makers towards goals set out for the different objectives, and their relative importance. GP has gained considerable attention and its use is wide-spread, see Romero (1991), Ignizio (1976), Schniederjans (1995), Tamiz et al. (1995) and Aouni and Kettani (2001) and Jones and Tamiz (2010).

From its conception, GP has become recognized as an important mathematical approach for academic research. An establishment of a set of goals forms the basis for the formulation of a GP model. The target value for each goal should be seen as the Decision Maker's (DM) desired outcome. Negative and positive deviation variables have to be introduced that represent, respectively, the quantification of the under and over achievement of the goal from the target value. The purpose of GP is to minimise the unwanted deviation between the achievement and the target of the goals.

There are three basic variants in GP: lexicographic, weighted, and Chebyshev (minmax) GP, see e.g. (Jones and Tamiz, 2010). While lexicographic GP was dominating the research in the first four decades, weighted GP seems to have gained considerable ground. Consider a set of goals $K$. Weighted GP (WGP) considers all goals $i$ ($i \in K$) simultaneously and tries to minimise a weigthed sum of the unwanted deviations. The deviations are weighted according to the relative importance of each goal for the DM. The algebraic structure of the WGP model (with percentage normalisation) is:

$$\text{Min } z = \sum_{i \in K} [\frac{u_i n_i + v_i p_i}{b_i}]$$

subject to

$$f_i(x) + n_i - p_i = b_i, \ \forall i \in K$$

$$n_i, p_i \geq 0, \ \forall i \in K$$

$$x \in F \tag{2.4}$$

where $u_i$ and $v_i$ are the non-negative weights attached to negative ($n_i$) and positive ($p_i$) deviations from the target value $b_i$. The achieved values $f_i(x)$ are in linear GP functions of $x$, and $x$ is a set of decision variables to be determined. $F$ is a set of (optional) hard constraints. To eliminate bias towards the objectives with larger magnitude, the percentage normalization technique (Tamiz et al., 1998) introduces $b_i$ into the objective function (assuming $b_i > 0$, $i \in K$).

Chebyshev or minmax GP was introduced by Flavell (1976). In this approach, the maximum deviation from any goal is minimised. Let $\gamma$ denote the maximal deviation from amongst the set of goals, then the Chebyshev GP (with percentage normalisation) has the following algebraic format:

$$\text{Min } z = \gamma$$

subject to

$$f_i(x) + n_i - p_i = b_i, \ \forall i \in K$$

$$\frac{u_i n_i}{b_i} + \frac{v_i p_i}{b_i} \leq \gamma, \ \forall i \in K$$

$$n_i, p_i \geq 0, \ \forall i \in K$$

$$x \in F \tag{2.5}$$

Surveys of the literature (Jones and Tamiz, 2002) find little practical use of the Chebychev GP variant, although Ignizio (2004) presents an interesting application for the allocation of maintenance technicians. This is somewhat surprising, as it has several practical advantages, as outlined in Jones and Tamiz (2010). Chebyshev GP has the potential to achieve the most appropriate solutions for which the balance between the levels of satisfaction of the goals is needed. This is in part also because it is the only major variant that can find solutions to linear models that are not located at extreme points in decision space. Jones and Tamiz (2010) conclude that there should be a large number of application areas, especially those with multiple decision makers each of whom has a preference to their own subset of goals that they regard as most important. In the context of this dissertation, we consider it a suitable method for finding a solutions that are (close to) egalitarian or fair.

A more recent development in GP is the combination of all three basic GP variants into one GP decision model, thereby allowing the inclusion of the three underlying philosophies or lexicographic ordering, satisficing, and balancing. It was first introduced in Romero et al. (1998) in the context of showing that GP can be seen as the universal underlying framework for both compromise programming and the reference point method. The formal introduction of the extended GP approach is due to Romero (2001). The use of the Chebyshev metric is here more prominently present. The combination of WGP and Chebyshev GP in particular is useful to investigate the balance between efficiency and balance in a decision problem context. Consider a set $K_w \subseteq K$ of goals to be weighted, and a set of goals $K_b \subseteq K$ to be balanced. These sets do not have to form a proper partition of $K$. The basic version of the extended Weighted-Chebyshev GP (with percentage normalisation) then has the following algebraic format:

$$\text{Min } z = \alpha\gamma + (1-\alpha)\sum_{i \in K_w}\left[\frac{u_i n_i + v_i p_i}{b_i}\right]$$

subject to

$$f_i(x) + n_i - p_i = b_i, \ \forall i \in K$$

$$\frac{u_i n_i}{b_i} + \frac{v_i p_i}{b_i} \leq \gamma, \ \forall i \in K_b$$

$$n_i, p_i \geq 0, \ \forall i \in K$$

$$x \in F$$

$$0 \leq \alpha \leq 1 \tag{2.6}$$

The use of GP as a decision making tool in multi-objective programming has gained its popularity in assisting the decision-making process in many areas. Some illustrative examples are outlined below.

In quality function deployment (QFD) that starts with the house of quality (HOQ), a robust evaluation method is needed to study the interrelationships between customer needs and product technical requirements (PTR) while determining the importance levels of PTRs in the HOQ. For that purpose, Karsak et al. (2003) integrated two decision making techniques, a zero-one goal-programming and analytic network process (ANP) in their studies. By integrating the methods, it can handle multiple goals including cost budget, extendibility, and manufacturability into the product design process for determining the PTRs.

The use GP method also found in supply chain area. Wang et al. (2004) integrated an analytic hierarchy process (AHP) and preemptive-goal-programming (PGP) based multi-criteria decision-making methodology to take into account both qualitative and quantitative factors in supplier selection. In this research, the AHP process matches product characteristics with supplier characteristics (using supplier ratings derived from pairwise comparison) to qualitatively determine supply chain strategy, while PGP, mathematically determines the optimal order quantity from the chosen suppliers. Since PGP uses AHP ratings as input, this integration is believed to put greater emphasis on the AHP progress as the variation of pairwise comparisons in AHP will influence the final order quantity. Therefore the accuracy of supplier ratings can be ensured. A fuzzy goal programming approach is applied in Kumar et al. (2004) for solving the vendor selection problem with multiple objectives, in which some of the parameters are fuzzy in nature. A vendor selection problem has been formulated as a fuzzy mixed integer goal programming vendor selection problem that includes three primary goals: minimizing the net cost, rejections and late deliveries subject to realistic constraints regarding buyer's demand, vendors' capacity, vendors' quota flexibility and etc. This approach has the capability to handle realistic situations in a fuzzy environment and provides a better decision tool for the vendor selection in a supply chain. The fuzzy GP approach also has been applied to portfolio selection study such as in Parra et al. (2001). The portfolio selection is considered as usual multi-objective problem deal with the optimum portfolio for a private investor with three criteria to be taking into account: return, risk and liquidity . These criteria are not crisp from the point of view investor therefore been treated in fuzzy term.

The GP methodology also been applied for allocating resources in hospitals (Blake and Carter, 2002) by developing a model that allow decision makers to set case mix and case costs in such a way that the institution is able to break even, while preserving physician income and minimizing disturbance to practice. The case mix and cost models promote an equitable allocation of resources by allowing decision makers to explore an institution's production possibility frontier. Through this process, the impact of different practice and case mix strategies can be identified and the tradeoffs between cost, volume and clinical necessity can be explicitly determined. Within the same area, a computerized nurse-scheduling model (Azaiez and Al Sharif, 2005) is developed through a 0-1 linear goal program to improve the current manual-made schedules. The developed model accounts both for hospital objectives and nurses' preferences, in addition to considering some recommended policies. The implementation of the model in an experimental phase for six-month period is considered to perform reasonably well, based both on some quality criteria and feedback obtained from the survey.

The GP approach, in fact can be use for decision aiding in partner selection. Hajidimitriou and Georgiou (2002) present a quantitative model, based on the goal programming technique, which use appropriate criteria to evaluate potential candidates and leads to the selection of the optimal partner in International Joint Ventures (IJVs). In IJVs, the selection of the appropriate partner constitutes one of the major factors of success for the IJVs.

Calvete et al. (2007) investigate the use of GP to solve the vehicle routing problem (VRP). In VRP, besides a hard time window associated with each customer, defining a time interval in which the customer should be served, managers establish multiple objectives to be considered, like avoiding underutilization of labour and vehicle capacity, while meeting the preferences of customers regarding the time of the day in which they would like to be served.

An enumeration-followed-by-optimisation approach is proposed to solve this problem. Computational results show that this approach is adequate for medium-sized delivery problems.

## 2.7 Concluding Remarks

This chapter reviewed scientific theories on human behavior that are of relevance to better understand the concepts of rationality and fairness in the context of human cooperation.

Neuroscience and anthropology bring to the forefront a systematic and rather universal way towards human thinking (making decisions) and behaviour. An important concept is efficient biological computation, or the tendency to always and automatically associate a value to calculations (thinking). This is used to aim for the best long term results from the least immediate investment. Having a set of (implicitly defined) goals is crucial for finding these values. These goals can change over time, and be about short or long term goals. The 'utility' of an outcome or action therefore depends on the individual's state of mind when taking a decision. A collection of reward error prediction signals is used to measure the desirability of any real or imagined action or state, and guide us towards making biologically efficient decisions. The goals system itself does not need to be explicitly defined, but may be composed of in part from subconscious desires. This naturally means that the field of psychology is also important for understanding decision making.

From the Universal People hypothesis, we see that we all have an intuitive psychology for understanding other people's intentions or desires, an intuitive idea of economics to exchange goods or services, and a mental database for keeping track of who did what to whom, when, and why. This enables us to implement 'tit-for-tat' strategies about fairness over time, and leads to the concepts of mutualism and altruism. Most humans tend to punish cheaters and reward cooperation in a wish to enforce social norms or do good for themselves. Next to sham-emotions to try to influence other people's opinions about yourself, and self-deception to retain a positive self-image, feelings of altriusm can be genuine but there can also be genuine differences in our beliefs about what is fair in cooperation.

Limits to rationality can in part be attributed to our limited ability to reason about complex multi-valued problems, and may be further influenced by cognitive bias. Emotions and how we feel about altruism, however, also seem play a role. Social dilemma studies distinguish between cooperative, competitive, and invididualistic attitudes. Models of cooperation have incorporated asymmetry or heterogeneity in preferences about fairness, and stress the importance of the mental model we construct about the type of players we face. Models of fairness can be classified as incorporating procedural fairness, distributional fairness, or reciprocal kindness. Finally, the option of punishment is also important, as in the absence of punishment, cooperation may not flourish.

# 3

# Cooperative Game Theory

## 3.1 Introduction

Several classical solution concepts from cooperative game theory are reviewed. The problem to solve is how to divide the worth of a coalition to its members (or players). Numerical examples for the Drug Game and Land Game are presented, and used to investigate in which ways fairness is implemented (or not).

## 3.2 Cooperative Game Theory

Cooperative game theory, introduced in 1944 by von Neumann and Morgenstern, is the field in game theory that studies games in coalitional form and is based on the characteristic function of the game (Tijs, 2003).

Let $N$ be the set of players, then the characteristic function $v(.)$ for a game with transferable payoffs specifies $v(S)$ for every subset $S \subseteq N$ as the total payoff that the members of $S$ can be sure of receiving if they act together as a coalition (against all non-included members). Formally:

**Definition.** *A cooperative game theory in characteristic function form is an ordered pair* $< N, v >$ *where* $N$ *is a set of players* $\{1, 2, .., n\}$ *and the characteristic function* $v(.)$*:*$2^N \rightarrow R$*, with* $v(\emptyset) = 0$*.*

The value $v(S)$ thus represents the worth or payoff of coalition $S$ in the game $< N, v >$. This value $v(S)$ is assumed to be distributable in any desirable way amongst the members of $S \subseteq N$.

In many games $v(.)$ is superadditive, which means that for any two disjoint coalitions $S$ and $T$ ($S \cap T = \oslash$, $S \subseteq N$, $T \subseteq N$), the following inequality holds:

$$v(S) + v(T) \leq v(S \cup T) \tag{3.1}$$

It is then in players' joint interest to form the grand coalition $N$. The issue is then how to divide $v(N)$ between the players such that no coalition $S \subset N$ has the desire to split from the grand coalition. It is thus in the joint interest of the players to establish whether there exists a payoff distribution vector $X(N) = (x_1, ..., x_i, ..., x_n)$, which specifies how much each member $i \in N$ receives, that can induce each member to remain in $N$. Desirable properties for $X(N)$ include Pareto optimality:

$$\sum_{i \in N} x_i = v(N), \tag{3.2}$$

and individual rationality:

$$x_i \geq v(\{i\}), \quad \forall i \in N. \tag{3.3}$$

$X(N)$ is called an imputation if it has both properties, and a pre-imputation if only Pareto optimality holds. Due to the uncontroversial nature of these conditions, the classic solutions approaches for finding a suitable payoff vector $X(N)$ are restricted to the class of imputation vectors only. The different approaches in cooperative game theory then each offer different solution concepts for this problem. We will review the Core, Shapley Value, Bargaining set, Kernel and Nucleolus.

### 3.2.1 The Core

Imputations that satisfy 'subgroup' rationality:

$$\sum_{i \in S} x_i \geq v(S), \forall S \subset N, \mid S \mid > 1 \tag{3.4}$$

are called stable. The core $C(v)$ of the game (Shapley, 1952) is the set of all stable imputations. Let $X(S)$ denote a payoff vector listing only the payoffs of $X(N) = (x_1, .., x_i, .., x_n)$ that each of the members $i \in S \subseteq N$

receives, then formally:

**Definition.** *The Core (Gillies, 1959) of the game is the set of all imputations that cannot be improved upon by any coalition:*

$$C(v) \equiv \{X(N) \in R^n | X(N) = v(N), X(S) \geq v(S), \forall S \in 2^N\} \tag{3.5}$$

The core can be empty, but in case it is not then it generally consists of many points and makes no distinction between points. Unfortunately, only some types of games have a structure under which the game is guaranteed to have non-empty cores, independent of the particular parameter values of the instance. If the core of a game is empty, then one potential approach to build a stable grand coalition through a binding contract is based on the $\epsilon$-core. Given some real value $\epsilon$, the strong $\epsilon$-core $C_\epsilon(v)$ is (Shapley and Shubik, 1966):

$$C_\epsilon(v) \equiv \left\{X(N) \in R^n \mid X(N) = v(N); X(S) \geq v(S) - \epsilon, \forall S \in 2^N\right\} \tag{3.6}$$

The strong $\epsilon$-core is the set of pre-imputations where no coalition can improve its pay-off by leaving if it then has to pay the penalty $\epsilon$. For games with non-empty cores, the smallest feasible value of $\epsilon$ is zero, and then the solution concept is the same as the core. For games with empty cores but where a binding agreement can be made with all parties, this approach might be suitable as for some large enough value of $\epsilon$, solutions will exists that can ensure coalitional stability by binding agreement. An alternative description, the $\epsilon-$ core, is based on replacing $v(S) - \epsilon$ by $v(S) - \epsilon|S|$ in Eq.(3.6), and interpreting $\epsilon$ as the penalty for each player upon leaving the grand coalition.

To prove that the core of a game is empty or not is in general an NP-complete problem (Deng and Papadimitriou, 1994). Non-emptiness of the core for any profit game can, in principle, be established by solving the following linear program:

$$\text{Min } z = \sum_{i \in N} x_i \tag{3.7}$$

subject to

$$\sum_{i \in S} x_i \geq v(S) \quad \forall S \subseteq N \tag{3.8}$$

$$x_i \geq 0, i \in N \tag{3.9}$$

Clearly, the core is non-empty if and only if the optimal value of this program is $v(N)$, in which case any optimal solution belongs to the core. This program has an exponential number of constraints $(2^N - 1)$. Alternatively, taking the LP dual of this program, with dual variables $\lambda_S$, $S \subseteq N$, gives:

$$\text{Max } z' = \sum_{S \subseteq N} v(S)\lambda_S \tag{3.10}$$

subject to

$$\sum_{S \ni i} \lambda_S \leq 1 \quad \forall i \in N \tag{3.11}$$

$$\lambda_S \geq 0, S \subseteq N \tag{3.12}$$

The classic result known as the Bondareva-Shapley theorem, gives a necessary and sufficient condition for a game to have a non-empty core. It is based on the concept of balanced sets:

**Definition.** *[Balancing condition.] A subfamily $\mathcal{B}$ of $N$ is balanced if there is a set $\{\lambda_S | S \in \mathcal{B}\}$ of non-negative real numbers called balancing coefficients of $\mathcal{B}$ such that $\sum_{S \in \mathcal{B}, \mathcal{S} \ni i} \lambda_S = 1$, $\forall i \in N$.*

**Definition.** *A game $< N, v >$ with transferably utility is balanced if for any balanced family $\mathcal{B}$ with balancing coefficients $\lambda_S$, $S \in \mathcal{B}$,*

$$\sum_{S \in \mathcal{B}} \lambda_S v(S) \leq v(N) \tag{3.13}$$

The Bondereva-Shapley theorem can then be stated as follows:

**Theorem.** *[Bondareva-Shapley] The core of a game $< N, v >$ with transferably utility is non-empty if and only if $< N, v >$ is balanced.*

### 3.2.2 The Shapley Value

The Shapley value (Shapley, 1953) is one of the most interesting solution concepts in cooperative game theory which has drawn much attention (Tijs and Driessen, 1986). The main advantage of the Shapley value is that it assigns a unique payoff vector based on marginal contribution. Its allocation is one of many game-theory solutions which does not depend on the existence of the core.

**Definition.** *: [Kalai, 2008]The Shapley value of a transferably utility game $v$ is the payoff allocation $\varphi(v)$ defined by*

$$\varphi(v) = \sum_{S:i \in S} \frac{(|S|-1)!(|N|-|S|)!}{N!}[v(S) - v(S \backslash i)] \qquad (3.14)$$

This expression describes the expected marginal contribution of player $i$ to a random coalition in a random order. When player $i$ arrives and joins the coalition of earlier arrivers $S$, he is paid his marginal contribution to that coalition, i.e., $v(S \cup i) - v(S)$. His Shapley value $\varphi(v)$ is the expected value of this marginal contribution when all orders of arrivals are equally likely. The solution in the Shapley value incorporate fairness by referring to these four axioms:

- Efficiency: the solution should distribute the maximal total payoff.
- Symmetry: the payoff for every player should based on his contribution (input).
- Dummy player: any player who contribute nothing to coalition should obtain his value.
- Additivity: adding the solution of two games together produces the solution of the sum of these games.

According to Kalai (2008) the Shapley Value is not always in the core of the game but if the game is convex, meaning that

$$v(S \cup T) + v(S \cap T) \geq v(S) + v(T) \tag{3.15}$$

for every pair of coalitions $S$ and $T$, then the Shapley Value and all the n! profiles of marginal contributions (obtained under different orders of arrival) are in the core. The axiomatic properties of the Shapley value, its probabilistic interpretation as the mean contribution, and its balanced contributions property are reasons for its attractiveness as a fairness standard. However, the lack of a total ordering constraint in some of the problems in decision making would present serious problems to the application of the Shapley Value solution. For such kind of problems, a solution concept that does not depend upon total orderings is needed.

### 3.2.3 The Bargaining Set

The core solution of cooperative game theory assumes that the outcome of a game will be an imputation e.g the players will divide the full grand coalition. It is possible, however, to imagine that a sub-coalition could be formed, dividing only the worth of the sub-coalition. Group rationality would then argue for the extra to be divided amongst the other player not in this sub-coalition. In practice, players do not always do that, perhaps because those extra units might not be worth the effort of further negotiations. Aumann and Maschler (1964) proposed a solution concept for this kind of situation known as the theory of bargaining sets.

Bargaining is an important concept in cooperative game theory. It involves a bargaining procedure and discusses if there is a final deal or a breakdown. When a deal is made at the end, it means that this game has a cooperative deal, and the problem is formulated as a cooperative solution. However, if it comes down to a breakdown, the game will not lead to cooperation.

The simplest bargaining problem involves only a single coalition when the situation is formalized by specifying a set of players, a set of attainable utilities, and a disagreement point (the vector of utilities of the outcome that will result if the agents cannot come to agreement) (Bennett and Zame, 1988).

**Definition.** *[Aumann and Maschler] Let $< N, V >$ be a game with sidepayments and let (z, T) be an outcome for $< N, V >$ (Thus, $T$ is a partition of N, $z \in R$ and for each coalition $s \in T$, $z(S) = \sum z_i \leq v(S)$. If player i and j belong to the same coalition $s \in T$, an objection of i against j is a pair (w, U) where U is a coalition containing i but not containing j and $w \in R$ is a payoff distribution for U ($w(U) \leq v(U)$) that satisfies:*

$$w_i > z_i \tag{3.16}$$

$$w_k \geq z_k, \quad \forall k \in U \tag{3.17}$$

Aumann and Maschler's idea works for any coalition structure in any game. For a game in characteristic function form, a payoff, $x = (x_1, .., x_n)$ is rational for a coalition structure if

$$x_i \geq v_i, \quad \forall i \tag{3.18}$$

$$\sum x_i = v(S_j), \quad \forall j \tag{3.19}$$

Notice that in general $x = (x_1, .., x_n)$ is not an imputation. However we could say that $x = (x_1, .., x_n)$ is stable for the coalition structure $x = (S_1, .., S_k)$ if every objection can be met by a valid counter-objection. Let consider two players $i$ and $j$ in the same coalition $S_j$. Player $i$ has an objection against $j$ if there is some coalition $S \in N$ and payoff $y = (y_1, .., y_n)$ such that:

- S contains $i$ but not $j$ (Player $i$ threatens to form $S$ and leave out $j$)
- $y_k > x_k, \forall k \in S$ (all members of $S$ prefer $y$ to $x$)
- $\sum y_k = v(S)$ ($S$ can assure its member what is proposed in $y$)

Player $j$ has a valid counter-objection against $i$ if there is some other coalition $T$ and payoff $x = (z_1, .., z_n)$ such that:

- $T$ contains $j$ but not $i$
- $\sum z_k = v(T)$ ($T$ can assure its member what is proposed in $z$)
- $z_k \geq x_k, \forall k \in T$ (all members of T like $z$ at least as much as $x$)

The core with empty, unique or infinite solutions leaves room on the negotiation table. Maschler (1976) pointed out that in these cases, players are not helpless. For one thing, each of these players realize that without him, the rest of the players are worth less than with him, and therefore he is unlikely to settle for zero. He argued that the bargaining solution gives a more intuitive solution than the core.

The situation of two games described above led researchers to study bargaining within a coalition. Although there are several versions of bargaining sets, we focus in this research only the simplest one. The bargaining sets consider every possible coalition structure in the games. In each structure, the division of payoffs will be decided from bargaining among the players. The idea is to study every coalitional structure in a coalitional break-up of the group or from the collective rationality condition in the core. However, it is possible to imagine that a subgroup coalition would form, leaving other players out and dividing only the worth of that subgroup coalition. A possible explanation for this situation is the extra units might be felt not worth to the effort of extra negotiation. This situation may happen in business when time for example is very valuable. The idea of bargaining sets is quite impressive for solving coalitional game theory problems, however its solution concept also suffers from and underdetermined or empty outcome if the result is a breakdown.

When the core is empty, voting and bargaining theories focus on the different predictions that could be derived from, for example, the different institutional rules observed in reality (positive approach) or rules that are conceivable (normative approach) for such bargaining situations (Frechette et al., 2005). However, these issues are especially relevant in government formation bargaining problems in which the potential heterogeneity of bargaining power across group members has been studied.

For the purpose of this research, it is to be noted that there is no such study based on profit (monetary) between coalitions. It is believed that many bargaining situations in the real world do not fit perfectly with the theoretical bargaining models, for they are settled by arbitration. Kalai (2008) in their paper highlight the need to explore incomplete information about the feasible payoffs of different coalitions as it is not nearly as developed as its strategic counterpart. According to Lopomo and Ok (2001), most authors today appear to agree on at least three major robust, yet unexpected, empirical regularities that arise in bargaining games. First, proposed division accumulate around the 50-50 division; the actual outcomes are more fair than the usual prediction. Second, rejections, which should never be observed on the equilibrium path, occur in significant numbers. Third, more often than not, subjects who reject an offer make a disadvantageous counteroffer. These observations are in contrast with the standard game-theoretic predictions. Lopomo in his study states that one way of interpreting these findings is to argue that pure expected profit maximization cannot be the only criterion guiding the choices of the players in bargaining games. For instances studies from Ochs and Roth (1989) and Bolton (1991) have shown that fairness may be influence players' behavior. Bolton has proposed that fairness may guide a player's behavior when he get less than the opponent.

### 3.2.4 The Kernel and Prekernel

The kernel consists of those imputations for which no player outweighs another one.

**Definition.** *[Curiel, 1997]: The kernel K(v) of a game v is defined by*

$$K(v) \equiv x \in I(v) \mid s_{ij}(x) \leq s_{ji}(x) \tag{3.20}$$

*or*

$$x_j = v(j), \quad \forall i, j \in N \tag{3.21}$$

**Definition.** *[Curiel, 1997]: The prekernel PK(v) of a game v is defined by*

$$PK(v) \equiv x \in PI(v) \mid s_{ij}(x) = s_{ji}(x) \tag{3.22}$$

*or*

$$x_j = v(j), \quad \forall i, j \in N \tag{3.23}$$

If a payoff vector $x$ has been proposed in the game $v$, player $i$ can compare his position with that of player $j$ by considering the maximum surplus $s_{ij}(x) \equiv \max e(S, x)$. The maximum surplus of $i$ against $j$ with respect to $x$ can be regarded as the highest payoff that player $i$ can gain (or the minimal amount that $i$ can lose if $s_{ij}$ is negative) without the cooperation of $j$. Player $i$ can do this by forming a coalition without $j$ but with other players who are satisfied with their payoff according to $x$. Therefore, $s_{ij}$ can be regarded as the weight of a possible threat of $i$ against $j$. If $x$ is an imputation then player $j$ cannot be threatened by $i$ or any other player when $x_j = v(j)$ since $j$ can obtain *v(j)* by operating alone. We say that $i$ outweighs $j$ if $x_j > v(j)$ and $s_{ij}(x) > s_{ji}(x)$.

### 3.2.5 The Nucleolus

The Nucleolus proposed by Schmeidler (1969) offers single imputation solution which is similar to what the Shapley Value does. The difference is, however, that the Nucleolus solution is based on concepts of bargaining while the Shapley Value is based on axioms embodying a concept of fairness. The Nuleolus always consists of one point which is an element of the kernel and is in the core whenever the core is non-empty. Recall that the core consists of all imputation $x = (x_1, .., x_n)$ which satisfy $\sum x_i \geq v(S)$. For some games, no imputation satisfies all of these constraints hence the core is then said to be empty. This will create an unstable solution as each coalition can be violated by each other. However it could try to satisfy them as nearly as possible. One way to interpret this would be to make the largest violation as small as possible and this what the nucleolus can do. For every imputation x, define the excess of S at x, by

$$e_s(x) = v(S) - \sum x_i \tag{3.24}$$

We could think of this as a measure of the unhappiness of $S$ with $x$. This kind of solution tries to make the unhappy coalition as small as possible. Note that $x \in C(v)$ if and only if $e(S, x) \leq 0$ for all $S \subset N$ and $e(N, x) = 0$. Interestingly, the nucleolus seems to give a better solution for nonempty core problem as well. In other words, the nucleolus will find a point which is as far inside the core as possible. Unlike the Shapley Value, it is always in Aumann-Maschler bargaining set for the grand coalition.

One justification for considering the nucleolus as a solution, at least when the core is empty, comes from the considerations of stability. To get the imputation as the payoff, we need to have the grand coalition: all players must cooperate. If some subset $S$ of players have too large an excess, they would be strongly tempted to go off and do better for themselves, thereby breaking up the grand coalition. To have the best hope of keeping the grand coalition together, we should make the unhappy subset $S$ as little unhappy as possible. In some sense, the payoff distribution can minimizes the protests of all coalitions.

$$Maximize, z = \epsilon \tag{3.25}$$

subject to

$$\sum_{i \in S} x_i + \epsilon \leq V(S) \tag{3.26}$$

$$x \in X \tag{3.27}$$

We have seen this approach before (the $\epsilon$-core). Another solution that could solve the empty core problem is the Nucleolus. If the core is empty, no imputation satisfies all of these constraints. The Nucleolus solution suggests that these constraints could be satisfied as nearly as possible, hence the unhappiness of every coalition could be minimized in order to avoid the players breaking up.

In conclusion, the attractiveness of the nucleolus' solution is that it always exists and is unique, is in the Aumann-Maschler bargaining set, and is in the core if the core is non-empty.

## 3.3 Numerical Examples

The previously introduced solution concepts are applied to two particular games, the drug game and land game, described in Winston (2004). We apply the solution concepts of the Core, Shapley Value, Bargaining Set and Nucleolus (The kernel solution is not been discussed as its solution is closely related to the Nucleolus). These examples illustrate some of the difficulties that fairness concerns bring to the interpretation of these classic solutions. The two games will be revisited in Chapter 6 for testing goal programming-based models of cooperation with fairness concerns.

### 3.3.1 Drug Game

In the Drug Game, player 1 has invented a new drug. However, player 1 cannot manufacture the drug on his own but has to sell the drug's formula to a production company. There are two choices available: player 2 or player 3. The lucky company will split a 1 million profit with player 1. The characteristic function is $v\{\emptyset\} = v\{1\} = v\{2\} = v\{3\} = 0, v\{1,2\} = v\{1,3\} = 1000000, v\{2,3\} = 0, v\{1,2,3\} = 1000000$. Let $X$ be the payoff for each player in the game.

#### 3.3.1.1 The Core Solution

For this game, let

$$X = \{x_1, x_2, x_3\}$$

and any $X$ will be an imputation if and only if

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_1 + x_2 + x_3 = 1000000$$

and will be in the core if and only if in addition:

$$x_1 + x_2 \geq 1000000$$

$$x_1 + x_3 \geq 1000000$$

$$x_2 + x_3 \geq 0$$

$$x_1 + x_2 + x_3 \geq 1000000$$

The unique core solution obtained for this game is $\{x_1, x_2, x_3\} = \{1000000, 0, 0\}$. It is apparent from this solution that the core emphasizes the importance of player 1. Player 1 has the power to trade-off the two other companies with each other to the extreme that neither will receive any payoff. It is surprisingly not realistic as either player 2 or player 3 can naturally claim a reward for joining in a coalition with player 1. They can appreciate player 1's power position in this case, but both realise that it would be unfair if player 1 would indeed claim all payoffs. Furthermore, player 1 itself cannot make the profit of 1 million on his own, and should realise that neither player 1 nor player 2 will eventually accept to take part in a coalition if they receive zero payoffs. Obviously, the outcome is unfair and the coalition formed in practice cannot have its payoff vector in the core.

The game can be formulated as consisting of two stages. In the first stage, player 1 would select one of the two other players. It is then no longer possible for player 1 to change its mind. Thus, the game in the second stage now consists of only two players, and the core of this game is again nonempty, but allows an infinite number of solutions. Indeed:

$$X = \{x_1, x_2\}$$

will be an imputation if and only if

$$x_1 \geq 0, x_2 \geq 0, x_1 + x_2 = 1000000$$

and will be in the core if and only if in addition:

$$x_1 + x_2 \geq 1000000$$

The core solution obtained is

$$\{1000000 - x, x\}$$

where $0 \leq x \leq 1000000$.

Straffin (1993) shows that the stable set for this drug game consists of the core point of the original 3-player game, together with a curve which runs from the core to the bottom of the imputation triangle. Straffin assumes this kind of stable set has a nice economic interpretation. However, we still have no solution of who player 1 should choose to form the coalition. By splitting up the problem in two stages, the second stage looses the negotiation power of player 1 completely. Also, the solution in the second stage will in practice lie in a region that is a proper subset of the core, as the extreme solutions for which payoffs are not fairly distributed amongst the two players will not be acceptable by one of the parties involved.

#### 3.3.1.2 The Shapley Value Solution

The Shapley value offers a unique solution. The method implies that player $i$'s reward should be the expected amount that player $i$ adds to the coalition $S$ made up of players who are present upon his arrival. Consider the drug game consisting of three players, player 1,2 and 3, there are six possible arrival orders as presented in the first column of table 3.1. For instance, $\{1, 3, 2\}$ represents the coalition of player 1,2 and 3 with player 1 being the first to arrive followed by player 2 and then 3. The second, third and fourth column of the table provides the worth of that player due to the order of their arrival in that coalition. The last row of the table presents the average of each player's profit of all the possible six arrival order.

Table 3.1: Shapley Value: Drug Game

| Coalition | Player 1 | Player 2 | Player 3 |
|---|---|---|---|
| $\{1,2,3\}$ | 0 | 1000000 | 0 |
| $\{1,3,2\}$ | 0 | 0 | 1000000 |
| $\{2,1,3\}$ | 1000000 | 0 | 0 |
| $\{2,3,1\}$ | 1000000 | 0 | 0 |
| $\{3,1,2\}$ | 1000000 | 0 | 0 |
| $\{3,2,1\}$ | 1000000 | 0 | 0 |
| | $\frac{400000}{6}$ | $\frac{100000}{6}$ | $\frac{100000}{6}$ |

Thus, the Shapley value recommends the unique solution $\{x_1, x_2, x_3\} = \{\frac{400000}{6}, \frac{100000}{6}, \frac{100000}{6}\}$. In this solution everybody receives a payoff. It shows that player 1 has been forced to provide profits for both other players, even if player 1 actually wishes to collaborate with only one of them.

If only two players were considered in the game, the Shapley value gives the unique solution of $\{1,2\} = \{500000, 500000\}$. It does not solve the problem of who player 1 should choose to cooperate with, and it is clear that the equal split of payoffs is not necessarily the solution that players agree upon.

#### 3.3.1.3 The Bargaining Set Solution

For the drug game, there are five coalition structures as shown in the first column of table 3.2. For example: the first coalition structure, $\{1\}$, $\{2\}$ and $\{3\}$, represents the case that each company does not form any coalition with the other players; $\{1,3\}\{2\}$ represents the case that player 1 and 3 join in the coalition leaving player 2 behind; the last coalition structure shows that all the players join together in a coalition. The second, third and fourth column list the worth of the corresponding coalition for player 1,2 and 3, respectively.

Table 3.2: Bargaining Set: Drug Game

| Coalition Structure | Player 1 | Player 2 | Player 3 |
|---|---|---|---|
| $\{1\}\{2\}\{3\}$ | 0 | 0 | 0 |
| $\{1,3\}\{2\}$ | 1000000 | 0 | 0 |
| $\{1,2\}\{3\}$ | 1000000 | 0 | 0 |
| $\{1\}\{2,3\}$ | 0 | 0 | 0 |
| $\{1,2,3\}$ | 1000000 | 0 | 0 |

Table 3.2 shows the stable coalition structure of the bargaining set. As can be seen, the bargaining set provides the same solution as the core in which the power of player 1 is emphasised. If there is any objection on this division, it cannot be met by a valid counter objection by player 2 nor 3 as the coalition of both players $\{2,3\}$ gives zero profit. It is again not a very intuitively appealing solution concept for this particular game.

#### 3.3.1.4 The Nucleolus Solution

Consider the characteristic function of the drug game of the seven possible coalitions as shown in the first column of table 3.3. The second column represents the worth of that coalition while the third column measures the 'unhappiness' or excess of that coalition, as explained in Section 3.2.5. The fourth and fifth column consider the possible payoff vectors between the players in drug game.

Table 3.3: The Nucleolus: Drug Game

| S | V(S) | e(x, S) | (500000,500000,0) | (1000000,0,0) |
|---|---|---|---|---|
| {1} | 0 | $-x_1$ | -5 | -10 |
| {2} | 0 | $-x_2$ | -5 | 0 |
| {3} | 0 | $-x_3$ | 0 | 0 |
| {12} | 1000000 | $-x_{12}$ | 0 | 0 |
| {13} | 1000000 | $-x_{13}$ | 5 | 0 |
| {23} | 0 | $-x_{23}$ | -5 | 0 |

Let us first consider the payoff vector of $\{500000, 500000, 0\}$ as shown in fourth column. From the table 3.3, it can be seen the coalition $\{1, 3\}$ has the largest excess that should be minimized. Large excess is indicated by the most positive value in that particular column. Next, the payoff vector of $\{1000000, 0, 0\}$ is being considered. As can be seen from the table 3.3, all excess in each coalition is less than or equal to zero. At this point, it is assumed that all the players are happy with the division. Therefore, the payoff vector of $\{1000000, 0, 0\}$ is suggested by the nucleolus solution for the drug game. Not surprisingly, the nucleolus concept gives the same solution as the core since the latter is unique and the nucleolus lies within the core if it is non-empty.

### 3.3.2 Land Game

In the Land Game, company 1 owns a piece of land and values the land at £10000. Company 2 is a subdivider who can develop the land and increase its worth to £20000 while company 3 is a subdivider who can develop the land and increase its worth to £30000. There are no other prospective buyers. For this game, let assume $X$ is the payoff for each company (player hereafter) in the game. The characteristic function is $v\{\emptyset\} = v\{2\} = v\{3\} = 0, v\{1\} = 10000, v\{1,2\} = 20000, v\{1,3\} = 30000, v\{2,3\} = 0, v\{1,2,3\} = 30000$.

#### 3.3.2.1 The Core Solution

For this game, let

$$X = \{X_1, X_2, X_3\}$$

and any $X$ will be in the core if

$$X_1 + X_2 \geq 20000$$

$$X_1 + X_3 \geq 30000$$

$$X_2 + X_3 \geq 0$$

$$X_1 + X_2 + X_3 \geq 30000$$

Thus, the core for this game is $\{20000 \leq X_1 \leq 30000, 0, 30000 - X_1\}$. There exists an infinite number of core solutions. Core solutions will guarantee player 1 a payoff of at least 20000 and a maximum of 30000, while player 3 will get the rest and player 2 always receives zero payoffs, or in other words will not be selected by player 1. The presence of player 2 is nevertheless important as it gives player 1 the power to demand for player 3 a minimum payoff of 20000, the value of the coalition that player 1 could form with player 2.

It can be argued that most real solutions will lie outside the core of this game, and such that player 1 receives at most 20000 unless player 3 is altruistic. Indeed, player 3 will not believe player 1's threat that by joining forces with player 2 it can claim 20000, as player 3 knows that this solution is not fair for player 2. Depending on its beliefs on how powerful player 1 can be in the negotiations with player 2, it estimates that payoffs of player 1 in the coalition with player 2 would be at some point in the neighborhood of say $20000 - y$, for $y \approx 5000$. It is then sufficient for player 3 to offer player 1 the payoff $20000 - y + z$, for some small $z < 5000$ in order to secure the project. It is also possible in practice that player 1 still chooses player 2 if player 3 underestimates this value of $y$. Only if player 3 is really desperate to have the project will it offer 20000 to player 1, but never more unless it is altruistic. In the latter case, it may still offer player 1 more than 20000 as a gesture of altruism and towards anticipated future rewards in further potential deals with player 1.

#### 3.3.2.2 The Shapley Value Solution

Considering the land game consisting of three players, player 1,2 and 3, there are six possible arrival orders as presented in the first column of table 3.4. For instance, $\{2, 3, 1\}$ represents the coalition of player 1,2 and 3 with player 2 the first to arrive followed by player 3, and then player 1. The second, third and fourth column of the table provide the worth of a player due to the order of their arrival in that coalition. The last row of the table presents the average of each player's profit over all possible arrival orders.

The shapley value gives the solution $\{\frac{130000}{6}, \frac{10000}{6}, \frac{10000}{6}\}$ for the land game. The solution suggests the grand coalition is to be formed. It is clearly an unrealistic solution.

If only two players are considered in the game, the Shapley Value solution for coalition $\{1, 2\}$ is $\{15000, 5000\}$ and for coalition $\{1, 3\}$ is $\{20000, 10000\}$. These solutions may be interpreted as egalitarian or fair for each of these two-

Table 3.4: Shapley Value: Land Game

| S | Player 1 | Player 2 | Player 3 |
|---|---|---|---|
| $\{1,2,3\}$ | 10000 | 10000 | 10000 |
| $\{1,3,2\}$ | 10000 | 0 | 20000 |
| $\{2,1,3\}$ | 20000 | 0 | 10000 |
| $\{2,3,1\}$ | 30000 | 0 | 0 |
| $\{3,1,2\}$ | 30000 | 0 | 0 |
| $\{3,2,1\}$ | 30000 | 0 | 0 |
| | $\frac{130000}{6}$ | $\frac{10000}{6}$ | $\frac{40000}{6}$ |

player games, and hints at $\{1,3\}$ as being the preferred choice for player 1, but does no longer reflect the power that player 3 has in bringing down the payoff of player 1 below the value of 20000 due to the fairness concerns in coalition $\{1,2\}$. It also does not allow player 3 to be altruistic in offering more to player 1 than 20000.

#### 3.3.2.3 The Bargaining Set

For the land game, there are five coalition structures as shown the first column of table 3.5. The first coalition structure, $\{1\}$, $\{2\}$ and $\{3\}$ represents each company does not form any coalition with the other players while $\{1,2\}\{3\}$ represents player 1 and 2 join in the coalition leaving the player 3 behind. The last coalition structure shows that all the players join together in a coalition. The second, third and fourth column presents the worth of corresponding coalition for player 1,2 and 3, respectively.

Table 3.5: Bargaining Set: Land Game

| Coalition Structure | Player 1 | Player 2 | Player 3 |
|---|---|---|---|
| $\{1\}\{2\}\{3\}$ | 10000 | 0 | 0 |
| $\{1,3\}\{2\}$ | 20000 | 0 | 10000 |
| $\{1,2\}\{3\}$ | 20000 | 0 | 0 |
| $\{1\}\{2,3\}$ | 10000 | 0 | 0 |
| $\{1,2,3\}$ | 20000 | 0 | 10000 |

Table 3.5 shows the stable coalition structure of bargaining set. As can be seen from the table, player 2 is powerless by getting zero payoffs in all coalition structures. This is because the worth of $\{1,2\}$ is less than the worth of $\{1,3\}$ and also the worth of $\{2,3\}$ is zero. This gives player 2 a difficult situation to make any objection against any other coalition. However, it is argued that the power of player 2 exists in the game since his presence contributes to the division of the worth of $\{1,3\}$. It is again not a very intuitively appealing solution concept.

#### 3.3.2.4 The Nucleolus

Consider the characteristic functions of the land game consisting seven possible coalition as shown in the first column of table 3.6. The second column represents the worth of that coalition while the third column measures the unhappiness or excess of that coalition. The fourth and fifth column consider the possible payoff vector between the players in land game.

Table 3.6: Nucleolus: Land Game

| S | V(S) | e(x, S) | (15000,0,15000) | (20000,0,10000) |
|---|---|---|---|---|
| {1} | 0 | $-x_1$ | -5 | -10 |
| {2} | 0 | $-x_2$ | 0 | 0 |
| {3} | 0 | $-x_3$ | -15 | -15 |
| {12} | 1000000 | $-x_{12}$ | 5 | 0 |
| {13} | 1000000 | $-x_{13}$ | 0 | 0 |
| {23} | 0 | $-x_{23}$ | -15 | 0 |

The nucleolus solution offers the same solution as the Shapley value for land game. Therefore the same issue arises that the grand coalition should form in order to increase the worth coalition. Let first consider the payoff vector of $\{15000, 15000, 0\}$ as shown in fourth column. From the table 3.6, it can be seen the coalition $\{1, 3\}$ has the largest excess, 5 that should be minimized. Large excess indicates by the most positive value in that particular column. Next, the payoff vector of $\{20000, 0, 10000\}$ is being considered and as can be seen from the table, all excess in each coalition is less than or equal to zero. At this point, it is assumed that all the players are happy with the division. Therefore, the nucleolus offers the solution of $\{20000, 0, 10000\}$ for the land game.

## 3.4 Concluding Remarks

This chapter has given an overview of solution concepts for cooperative games. Most cooperative games have a characteristic function that satisfies the property of superadditivity. Thus, the maximal possible payoffs are generated by the grand coalition and it is in the joint interests' of all players to form the grand coalition. All attention then goes to how to divide the payoffs of the grand coalition amongst its members. Cooperative game theory is payoff-centered. It is demanding in terms of interpretation of solutions. A characteristic function should come with a story describing the relationships between players. Therefore, while cooperative game yields payoffs, these payoffs often suggest actions.

For games with non-empty cores, such as the Drug and Land games, superadditivity holds but nevertheless a practical solution would only imply the formation of a subcoalition. Applying the classic solution concepts of Core, Shapley, Bargaining, or Nucleolus prove difficult in that they generate solutions to which practical concerns related to fairness can be raised.

The classic approach for games with subcoalition formation is to apply cooperative game theory to any subcoaltion that then might be selected. This fails to incorporate the power that some players had in the grand coalition game, which will impact on the selection of the subcoalition that will form, and to which degree the payoffs are split in that subcoalition. Conversely, fairness concerns between the members of any subcoalition will have an impact on the choice of subcoalition in the first phase.

The classic solution concepts of cooperative game theory require a particular kind of substantive rationality of the decision makers. The power of an individual in any subcoalition is solely derived from the power it has of being able to take part in other subcoalitions. In order to find more realistic models, 'symmetry' or 'exchangeability' in the formulation of cooperative games is perhaps an assumption that should no longer be taken for granted. Players do think 'outside the game' about the strategic value of cooperation with certain other players with respect to future opportunities, and are concerned about the fairness of the allocation of a subcoalition's total worth amongst its players.

# 4

# Neuroscience, Goal Programming and Fairness

## 4.1 Introduction

After reviewing the essential ingredients of GP in Section 2.6, this chapter continues drawing parallels between GP and elements of decision making put forward in the field of cognitive neuroscience. This gives rise to a framework of GP modeling approaches that will be used in later chapters to model games of coordination where fairness is an explicit component of the rationality concerning the decision making of players.

## 4.2 Goal-based Cognitive Neuroscience

A comprehensive book on cognitive neuroscience for non-specialists is by Read Montague (Montague, 2007). A human brain is a biological computer performing calculations when making decisions. In contrast to silicon computations, biological computations automatically carry an extra measure of the value of that computation to the overall success or fitness of the organism. This efficiency measure drives us towards aiming for the best long-term returns from the least immediate investment. Having goals, expressing that we desire some things more than other things, is essential to gauge the worth of calculations. At the moment of taking a decision, the different short and long term goals we have in mind (implicitly) define a goal state. The goal state is thus multi-valued and with each goal a 'critic' is associated. Taking decisions involves building mental models of possible actions we or relevant others may take, each leading to different states. These models are based on experienced, actual, or imagined situations associated with each of these scenarios. The desirability of each scenario/state is measured by a value proxy scheme made up of the collection of error-prediction signals (e.g. changes in dopamine levels released in the brain) that is a composite of the signal of each critics' comparison of the particular state with the goal state. The mind filters the importance of each critic's direct signal, reflecting the relative importance of the particular goal or criterion. The effects of changes in this collective weighted error signal, when thinking through different states, guides us towards the best state and actions to take. The goal state or goals itself, in particular for complex goals, do not need to be explicitly defined or be available as conscious experience. We rather follow the guidance signals. That our brain works this way is a direct result of the need for efficiency in biological computations. The fact that goals often change over time, and that some are subconscious, is one of the reasons why psychology is important in understanding decision making. It is therefore closely related to altruistic behaviour, to social cooperativity and to the cognitive process that guide behaviour in fields as diverse as economics.

## 4.3 A Goal Programming Approach

In this section, we formulate decision making based on GP while interpreting it using the concepts put forward in the field of cognitive neuroscience.

### 4.3.1 The Reward-Prediction Error Signal Goal Programme

Let $X$ be a (discrete) set of states we consider worthwhile examining at a time of making a decision, and $x \in X$ be any state in this set. We want to find the best state of $X$ to move to. The (imaginary) goal state $x^o$, that will often be beyond reach and thus not in $X$, is defined implicitly by a set of target values $g_i$ (numbers) we associate with $k$ different aspects or decision criteria, $i = 1, .., k$. If every aspect $i$ has a corresponding 'critic' in the mind that evaluates a state $x \in X$ by comparing the function value of the state $f_i(x)$ to $g_i$, the critic composes an immediate feedback or reward-prediction error signal $n_i + p_i$ based on their difference, but the relative importance of this immediate feedback signal is tempered by a relative weight $w_i$ that reflects the (long term) value of the error signal on this criterion for the decision maker. The simplest representation of a total valuation of a state $x$ would arguably be the linear function $\sum_{i=1}^{k}(w_i^n n_i + w_i^p p_i)$. If both positive and negative deviations are equally important, then $w_i^n = w_i^p$, whereas for one-sided goals, for example, either $w_i^n = 0$ or $w_i^p = 0$. The mind selects the best state to move to by examining the different states $x \in X$ and retaining the one for which the total valuation function is best. This can be captured by the following Weighted GP:

$$\min z(x) = \sum_{i=1}^{k}(w_i^n n_i + w_i^p p_i) \tag{4.1}$$

subject to

$$f_i(x) + n_i - p_i = g_i, i = 1, ..., n \tag{4.2}$$

$$x \in X \tag{4.3}$$

It is assumed in this model that all deviations are measured in the same neural currency (e.g. dopamine levels), and therefore there has been no need to normalize.

The above model is just the standard Weighted GP model, but the terms have been labelled as to allow the mapping of the approach to the general concepts that have been developed in cognitive neuroscience as discussed above.

### 4.3.2 Why Defining Goals Beyond Reach?

One important observation is that both the psychology/neuroscience community and the GP community find compelling reasons to define the goal state as beyond reach, but they seem to do so for different reasons. According to Gregory (2005) and Montague (2007), it is necessary in order to make sure that we keep the desire to learn. In the GP literature, the reason is to avoid ending up in solutions to the given problem that are not Pareto optimal, see Jones and Tamiz (2010). It is an open question whether there are psychological grounds on which to also consider the latter reason as important, but it seems reasonable to conjecture that it is. Indeed, from an evolutionary viewpoint, organisms that make their decision based on defining the goal state beyond reach will in general have a higher chance to arrive at non-dominated solutions than those that don't set ambitious targets, and thus the former organisms must have had an evolutionary advantage.

It is not claimed that GP is an accurate description of the actual decision making processes in the brain (for this purpose, models are typically based on artificial neural networks and fuzzy logic). At a high-level, however, the Goal Programming approach does seem to capture the logic put forward in Montague's book.

### 4.3.3 GP and Utility Function Theory

The difference between using goal programming instead of utility function theory might seem somewhat artificial, as there is a high level of mathematical equivalence between the two (Tamiz et al., 1998).

There is, however, a philosophical difference. Not only in neuroscience, but also in the field of psychological research and informing science in particular (Gill, 2008), it is argued that humans apply goal-based reasoning. Mathematically, this maps onto goal-based utility functions. Due to limitations in our capacity to simultaneously and explicitly consider different goals, and the fleeting nature of subconscious goals, the utility of a particular decision can be highly dependent on *the moment* the decision is taken, and can be influenced by framing, learning, or changes in circumstances.

The outcome of a decision is thus highly dependent on the the mindset of a player. The same decision might be perceived good at one time, but rejected at another time by the same player. Traditional utility theory rejects this possibility.

## 4.4 Modeling Fairness Concerns

### 4.4.1 Inequity Aversion Goal Programme

One of the challenges to model fairness is the lack of a proper universally agreed upon measure of fairness. According to Montague (2007), important recent work on fairness have come from the creative work of Ernst Fehr, showing that people are slightly asymmetric in their perception of fairness in relation to other players. A non-technical description would be that the utility of a player is reduced the more solutions produce unequal payoffs as follows (Montague, 2007):

$$P - \alpha \ \mathrm{Max}\{P - P', 0\} - \beta \ \mathrm{Max}\{P' - P, 0\} \tag{4.4}$$

where $P$ is a player's own payoff and $P'$ is the payoff of the other player, and $1 > \alpha > \beta$ are chosen constants. Based on experimental evidence from a series of economic-exchange games to probe the human instinct to be fair and to punish those who are not, the Fehr and Schmidt (1999) model allows asymmetry in fairness perception by different types of players. The model is known as the inequity aversion model in which inequity aversion means that people resist inequitable outcomes; i.e., they are willing to give up some material payoff to move in the direction of more equitable outcomes. A more technical treatment of this and other models will be conducted in later chapters.

What is important to note here is the structure in which fairness is modelled. Fairness is explicitly incorporated and compared directly with the (monetary) payoff. In terms of GP, we have thus the inclusion of fairness as one of the goals that players have in a Weighted GP model as outlined in Section 4.3. The approach will be used in Chapter 6 to reformulate the Drug Game and Land Game.

### 4.4.2 Theory of Mind and Chebyshev Goal Programme

While we do not dispute the experimental findings of Fehr and Schmidt nor their conclusions about asymmetry in fairness, there is a natural alternative to modelling fairness as in Section 4.4.1. This alternative is based on a more direct approach of applying the theory of mind (ToM) concept, and viewing fairness as an algorithm which aims to find solutions $x \in X$ that come as close to egalitarian as possible. It also allows the incorporation of asymmetry in fairness perception.

In games of coordination with two players, for example, a player needs to consider the other player's intentions and desires. A decision maker therefore develops a model of the other player in her own mind (ToM, or mentalizing as in Frith and Frith (2003)). This ToM model is arguably very similar in structure to her own model as expressed by Eq.(4.1) to Eq.(4.3). In the simplest approach, the state vector $X$ would be the same, as are the type and number of goals considered, but in general the goal state $x^{o'}$ will be different from her own, reflected in $g_i' \neq g_i$ $(i = 1, ..., k)$, and also the relative importance $w_i'$ of the error signals will differ. Let the total valuation function of the other player in her mind be $z'$.

In coordination games, a decision maker looks for a 'fair' compromise solution by considering both her own and the other player's desires. The following Chebychev GP is a simple approach to representing this:

$$\min \lambda \tag{4.5}$$

subject to

$$z(x) \leq \lambda \tag{4.6}$$

$$\gamma z'(x) \leq \lambda \tag{4.7}$$

$$x \in X \tag{4.8}$$

where $\gamma$ is a parameter representing the relative importance that the player assigns to the other player's desires, and $\lambda$ is a decision variable representing the maximum value for any of the evaluation functions. This approach thus also allows for the consideration of asymmetry in the perception of fairness by a proper choice of $\gamma$. When the player considers both total valuation functions of equal importance, then $\gamma = 1$. When the other player's function is considered of less (or no) importance, then $\gamma < 1$ (or $\gamma = 0$).

The other player can be modelled in a similar way. His own criteria lead to a similar model in structure, but naturally goals and weights may differ because each player might actually be wrong in her/his ToM model about the other player's true mindset (even if assuming that they consider the same set of criteria and the same set of states).

This approach offers a procedure for not only determining which offers are considered fair offers, and which not, but also whether cooperation can be established or not. For example, one player can use her model to determine the set of outcomes $x^* \in X$ for which $\lambda$ is minimised, or thus a fair compromise between her own desires and what she thinks are the desires of the other player. The other player can evaluate these offers in his own model, and accept when $z'(x^*) \leq z(x^*)$, and reject otherwise (with $\gamma = 1$).

The consideration of several criteria to determine your own payoff can be easily incorporated, as well as the acknowledgment that there might be differences in the way another player might think about these criteria and their relative importance.

Non cooperation in this approach can only occur when players differ in their ToM of each other, or when the value of $\gamma$ differs from 1. In other words, when players accept the other players model as from 4.1 to 4.3 as a true and honest reflection of their desires, and players value each others opinions equally ($\gamma = 1$), the set of fair solutions $x^* \in X$ that each of them identify through their own model as fair are equal. This doesn't need to imply that both their objective functions can reach the same minimum level, as for some problems solutions $x \in X$ may actually tend to provide higher benefits to one player than the other player (reflected in a high value of $\lambda$ and a large slack for one of the players' objective function). Still, fair solutions can still be identified and thus cooperation may still be achieved if players acknowledge the inevitability of this inequality! This is in contrast to the utility function approach outlined in Section 4.4.1. It seems much harder to establish whether cooperation is possible in Fehr and Schmidt's utility approach for problems where inequity aversion is inevitable.

The structure of the Chebyshev approach ties in well with the findings that cooperation is more likely when players can identify themselves as sharing the same social norms and are cooperative. It also ties in with the findings about the importance of knowing the type of player you are dealing with. This allows players to construct a ToM that is a close as possible to the real desires of the other player. Cooperation is then more likely for players that are all willing to act fair, as they would all choose $\lambda$ (close to) 1. Cooperation is unlikely, however, if some or all of the players are very competitive or individualistic, as players then choose $\lambda$ (close to) 0, limiting the overlap between what players identify as fair solutions within $X$.

An open question, however, is whether players try to build and thus acknowledge an accurate ToM of the other player, or a personal idealised version how they think they other player should behave. While the first would be arguably the best strategy for playing games of cooperation in which cooperation measures success, the many disputes about what is a fair solution to real coordination problems (e.g. local neighborhood disputes, taxes and benefits, environmental issues) indicate that the latter is perhaps more often true.

## 4.5 GP, Irreducible Uncertainty, and Deception

By considering different weight combinations and different goal values, the GP models presented in previous sections can capture the differences between individuals, but also the dependency of an individual's decision on the particular state of mind during the moment of decision making. The underlying cognitive model is that the decision maker sets goals or aspiration levels for the objectives under consideration, and then weigh up prospective alternatives through a dynamic and iterative comparison with the aspiration levels.

Montague (2007) argues that uncertainty will be especially relevant in new decision making situations but might reduce considerably for repeatedly played decision games. He further argues, however, that there should always be some uncertainty as an essential element in social coordination games to prevent players from being exploitable.

A non-zero level of unpredictability of the weights would be one way in which this concept of irreducible uncertainty can be modelled. Alternatively, or in addition, irreducible uncertainty may also arise from uncertainty about the actual goals values itself.

The GP models presented, and in particular the Chebyshev GP model of fairness, also offer a way to model how players may exploit information asymmetry by using deception. As seen in Section 2.5.3, players may exhibit a level of self-deception or try to envoke sympathy by exhibiting sham-emotions towards other players. In the Chebyshev GP model of fairness, this may result in players adopting a ToM model of the other player that will lead to more favourable outcomes for the other player. Self-deception may be a strategy to make the other player truly believe that you deserve outcomes more favourable for yourself.

## 4.6 Concluding Remarks

Goal Programming (GP) is widely applied to find practical solutions to many problems in Operational Research where decision makers have multiple goals to consider. The foundation of GP is the theory on satisficing by Herbert Simon, developed in the period between 1955 and 1960.

In this chapter, we are first to have identified striking similarities between the GP framework and the computational theory of mind (on how humans make decisions) developed in the field of cognitive neuroscience. While we do not wish to claim that GP accurately represents the real decision making processes in the brain, it does seem to capture at an abstract level some of the key concepts, including the concepts of goals and the multi-valued aspect of the goal state, efficient biological computation, theory of mind, and reward prediction error mechanisms.

Both GP and neuroscience find compelling reasons that humans who set their goals just out of reach fare better. The reason from neuroscience is that this is essential for humans in order to keep the desire to learn. In weighted GP, it is an advantage it offers a guarantee of achieving Pareto efficient outcomes (Romero et al., 1998). We have argued that the latter practical approach adopted in GP could be explained, at least from a conceptual point of view, as an essential characteristic to define fitness of an individual in a population, and thus must have had an evolutionary advantage. GP also offers a way to incorporate irreducible uncertainty in the context of cooperation between individuals.

While there are similarities between GP and classic utility function theory in a mathematical sense, the philosophies of satisficing and balancing underlying Weighted and Chebyshev GP offer advantages in showing the dependency of decisions on multiple goals and in relation to the state of mind of a person, and how this person thinks about the state of mind of other relevant players, and how important the latter consideration is when making decisions. We believe that a GP-based theory of fairness is less bound to pre-imposed concepts which outcomes are fair in a particular game in comparison to the utility based models such as the well-known inequity aversion model.

We have also identified the possibilities of the GP models to operationalise some concepts from the field of psychology, including the level of social identification amongst the players and the use of deception.

# 5

# A GP Approach to Model Fairness in Human Decision Making

## 5.1 Introduction

In this chapter, Goal Programming (GP) is applied to modelling the decision making processes in the well-known Ultimatum Game. The decision model for a player is a Chebychev GP model that balances one's individual desires with the mental model one has of the desires of other relevant players. In this approach, fairness is modelled as a universal mechanism, allowing players to differ in their belief of what a fair solution should be in any particular game. The model's conceptual framework draws upon elements considered of importance in the field of cognitive neuroscience, and results from the field of psychology are used to further specify the types of goals in the model.

Computer simulations of the GP models, testing a number of Ultimatum, Dictator and Double Blind Dictator Games, lead to distributions of proposals made and accepted that correspond reasonably well with experimental findings. The statistical analysis is then conducted to support the findings. Next, the types of goals distribution of accepting and rejecting the offers are analysed and their associations with the decisions are examined. A parallel is drawn between the UG and a common real-life situation to help explain the rationale of the model and the final section summaries the main conclusions.

## 5.2 Goal Criteria

This research posits that there are strategic and non-strategic goal criteria of players in making decisions in games. The non-strategic goal criteria is an expected outcome of the game of that player while strategic is an imagined criteria of the outcome of the other player. This model predicts that one player makes the best offer if these strategic and non-strategic goal criteria are being considered when making decisions. Hence, it could be hypothesised that the rejection rate in UG could be reduced if a proposer can make the best offer to a responder. The best offer must be interpreted with caution because what is best for that player might not be so for the other player. Rationally, the acceptance rate is increasing with the offer value. However, even low offers are accepted if they are the best offers to that particular player. The reasons for this to happen have been discussed in Chapter 2. Thus, it could be hypothesised that all criteria of goals influence the offer values from the proposer. The central question is: how does a player arrive at the his or her best strategy (offer, or accept/reject)? The interpretation of how a best solution is arrived at differs from the classic solution concepts in cooperative game theory as the traditional utility function is replaced by a reward-prediction error minimising mechanism, that draws its existence from a parallel made between GP and cognitive neuroscience. In addition, the strategic element of game theory is incorporated by using the concept of the mental model of the other player, and seeking a Chebyshev minimising solution. Four different goal criteria in the mind of each player are considered: (1) monetary pleasure, (2) fear of rejection, (3) concern about reputation, and (4) Emotion. The following general interpretations are assigned to these four criteria:

- **Monetary pleasure(MP)**. Players derive some immediate pleasure from the expected financial reward.

- **Fear of rejection (FoR)**. How badly does a participant want this particular game to succeed? A proposer in the UG may not wish to offer low values if he desperately wants this game to be successful, i.e. the offer be accepted. A responder may accept a low offer if his fear for rejection is high. This criterion can be generalised to decision making for companies. Companies can have a high fear of rejection whenever the current proposal to collaborate is important for survival of the company, or when it carries other significant strategic value (only realisable when this project goes ahead). On the other hand, fear of rejection could be low whenever there are plenty of other opportunities for economic success, or when the *search cost* for finding the next opportunity is considered low. Kravitz and Gunto (1992) in their study has used the term fear of rejection to explain anomaly in in ultimatum game.

- **Concern about reputation (CR)**. When proposing or accepting a certain deal in the UG, both proposer and responder send signals to their 'peer group' what type of person they are. The peer group can consist of the (imaginary) group of prospective players, or more generally, a society or authority imposing norms of expected behaviour that the players' wish to respect. When reputation is of high concern, proposers will not be overly greedy. They then want to give the message to other potential players that they are good to collaborate with, or to their society that they are confirming to established norms of behaviour. On the other side, proposers may also be reluctant to give too much away as they also do not wish to give the signal that they are easy to exploit. Responders, who typically receive less than half in the UG, will not want to accept offers when their concern about reputation is high, as they do not wish to signal to other prospective players that they are easy to exploit. On the other hand, they would not feel much remorse accepting high offers, as the peer group knows that it is not their responsibility to make the offer. In a more general economic context, companies that value their survival assign worth to their (brand) reputation. The representation of reputation to measure fairness in a game also supported by various researchers (Falk and Fischbacher, 2006, Nowak et al., 2000). The great influence of reputation criteria also stated in Fehr and Gachter (2002) studies where players make predictable strategic adjustments when it been incorporated into a game.

- **Emotion(EM)**. How well does one player trust and sympathise with the other player, or how badly does one want to punish the other player at his own monetary expense? Having sympathy means doing good now at your own expense to others you place your trust in, in anticipation of future rewards returned. In an artificially created game such as the single-shot UG where players do not know each other, sympathy and trust should not be of any importance. Evolutionary psychology, however, would suggest that it may play some role - we subconsciously think about potential future collaborations with this other player, and thus tend to consider the long term effect of our current behaviour of our future success (Gintis, 2000). On the other side on this coin, there is righteous anger, whereby a responder in the UG punishes greedy proposers at his own cost. When the UG is repeatedly played, proposers might also wish to punish greedy responders by not offering much. In addition, self-reported questionnaire of emotional states suggest that emotions such as shame, guilt or anger play an important role in decision making (Bosman and van Winden, 2002, Fehr and Fischbacher, 2004).

For the proposer model, monetary pleasure will show the highest value of goal that player wants to achieve followed by fear of rejection, reputation and sympathy. In contrast, the value of goal chosen in the responder model is in reverse order. To further elucidate the underlying neurocognitive process, the GP modeling is employed in UG, DG and DBDG.

## 5.3 Ultimatum game

The Ultimatum Game is reviewed in Chapter 2. Recall that the UG is a simple yet powerful game to demonstrates simple economic decisions can be influenced by fairness, emotions, and the desire to respect social norms. Given its importance in models of strategic behaviour, it has been studied widely through experimental methods. In the UG there are two players, one proposer and one responder who have to divide a fixed pie. Suppose there be an amount of 10 to be distributed between the proposer and responder, and let $x$ represents the amount that the proposer will keep for himself ($x \in [0, 10]$).

Both players will assign, in the context of the ultimatum game, a monetary goal $g_i$ as well as a relative importance $w_i$ to each of the above described goal criteria ($i = 1, ..., 4,\ g_i \in [0, 10],\ \sum_{i=1}^{4} w_i = 1$), thereby describing the imaginary of mental monetary goal $g_i'$ of the other player as well as a relative importance $w_i'$ to each of goal criteria, ($i = 1, ..., 4,\ g_i' \in [0, 10],\ \sum_{i=1}^{4} w_i' = 1$).

The proposer and responder will construct a GP model each in which some of the negative and positive deviational variables, $n_i$ and $p_i$ ($i = 1, ..., 4$), are minimised. The number of goals is expected to be eight for each GP model of proposer and responder. It is assumed that all players adopt the goal in the models, but the players can differ in the relative weight they assign to the error signals. For each GP model, there are 4! way of assigning relative weight in each monetary goal and also 4! way for each mental monetary goal of other player. To restrict the number of possibilities, four different and arbitrarily chosen levels are considered for each of the relative weight $w_i$ and $w'_i$: 0.44, 0.3, 0.18 and 0.08. The values chosen represent its relative importance towards the goals' criteria. The sum of weights is equal to 1 for normalization purposes. Considering that all possible combinations of weight values for each goal (4! ways of own goal and 4! ways of mental goal), there are 576 different offers from the proposers to responders and for responders himself, there are 576 accepted values that will be considered as accepted offers. Both models proposer and responder can be solved using Lingo programming (A.1.1 and A.1.2). By considering each proposer plays an ultimatum game to each different responder each time, the total game to be played is 331776. The collection of these games are solved on a normal computer using Visual Basic Programming in Excel (A.1.3) to see the distribution of accepted and rejected of proposer's offers.

### 5.3.1 Experiments

An experiment of dividing a fixed amount of 10 between proposer and responder is simulated on a computer to observe the acceptance rate of UG. Using this simulation result, the research finds support for model's prediction. A typical proposer might construct the following objective function and associated goal constraints for himself (corresponding to the model Eq.(4.1) to Eq.(4.3)). The values of monetary goal, $g_i$ $(i = 1, ..., 4, g_i \in [0, 10])$ are ranked values randomly chosen as the benchmark to represent the types of behaviour of the players. In some decision making systems some goals seem to prevail. Those goal values are varied by the relative weights attached to each deviation variable measuring its importance for each goal. The Chebyshev GP then provide the best solution by minimising the deviations from reaching the goals. For the first experiment, let is start with the following value for each goal.

$$min\ z(x) = w_1 n_1 + w_2 p_2 + w_3(n_3 + p_3) + w_4 p_4 \tag{5.1}$$

$$x + n_1 - p_1 = g_1 = 10 \tag{5.2}$$

$$x + n_2 - p_2 = g_2 = 7 \tag{5.3}$$

$$x + n_3 - p_3 = g_3 = 5 \tag{5.4}$$

$$x + n_4 - p_4 = g_4 = 4 \tag{5.5}$$

$$0 \leq x \leq 1 \tag{5.6}$$

For monetary pleasure, more is always giving more pleasure. Therefore the goal is set at 10 and the negative deviation from the goal is to be minimised. Fear of rejection implies that the proposer does not wish to keep too much for himself, and thus high values need to be penalised or in other words values above a goal of, say, 7, will make the positive deviational variable non-zero, and this is to be minimised. Concern about reputation works on two levels for the proposer: on the one hand, giving too much away would give a signal to others that she is easy to exploit, and on the other hand, keeping too much for himself would give a signal to others that she is not a very attractive party to play the game with. Therefore, the goal falls naturally somewhere in the middle, say at 5, and both positive and negative deviational variables are to be minimised. Finally, sympathy towards the responder may imply that the proposer would be generous in anticipation of returned favours in the future. A goal of 4, and minimisation of positive deviations, would reflect this. Although the goals chosen are constant for every player, it differs in weights attached to the goal which representing how one player prioritize that goal in games.

In addition, the proposer also builds a ToM model of the responder by considering the values of mental goal for each goal criteria. Note that the values of monetary goal, $g'_i$ $(i = 1, ..., 4, g'_i \in [0, 10])$ are in reverse ranking order and randomly chosen.The ToM model in the mind of a typical proposer might look similar like:

$$min\ z'(x) = w'_1 p'_1 + w'_2 p'_2 + w'_3 n'_3 + w'_4 n'_4 \tag{5.7}$$

$$(1 - x) + n'_1 - p'_1 = g'_1 = 0 \tag{5.8}$$

$$(1 - x) + n'_2 - p'_2 = g'_2 = 0.5 \tag{5.9}$$

$$(1 - x) + n'_3 - p'_3 = g'_3 = 3 \tag{5.10}$$

$$(1 - x) + n'_4 - p'_4 = g'_4 = 4 \tag{5.11}$$

$$0 \leq x \leq 1 \tag{5.12}$$

Recall that $1 - x$ is received by the responder if he accepts. The ToM model of the responder helps the proposer to find the minimum offer that the responder will accept. Goals and objective function can be explained as follows. Note that the deviational variable attached to each goal which representing under and over achievement of a goal may differ based on the nature of the goal criteria in ToM model. Monetary pleasure for the responder means that any money is good money and should not be rejected. Based on pure rational grounds from within the UG, any offer made by the proposer should thus be accepted. Thus, the goal for this criterion is 0, and the positive deviation should be minimised (because this criterion should drive the acceptance level to as low values as possible). A high fear of rejection implies that the responder will again strive to accept all possible offers made by the responder, even if they are fairly small. Thus, if the proposer thinks that the responder desperately needs the money, the goal for this criterion can again be set low, at say 0.5, and the positive deviation should be minimised.

Concern of reputation means that the responder would not be happy with too low offers, as this signals to others that the responder can be exploited. Low offers under say 3 would therefore cause error signals for the responder, and thus the negative deviation needs to be minimised. Positive deviations to this goal do not receive any emotional response, since the responder knows that he is then merely accepting generous offers and will not be looking as being too greedy by the social group. If the responder is sensitive to feelings of righteous anger at low offers, then the proposer anticipates that also these need to be kept under control. These feelings may start to arise at say 4, when there is a real difference between what the two parties would receive, and negative deviations need to be minimised.

With these functions in mind, the proposer now wishes to determine the offer that seems fair in his mind by the Chebychev GP given by Eq.(4.5) - Eq.(4.8), where $x \in [0, 10]$, which can be represented as follows:

$$\min \lambda \tag{5.13}$$

subject to

$$w_1 n_1 + w_2 p_2 + w_3(n_3 + p_3) + w_4 p_4 \leq \lambda \tag{5.14}$$

$$\gamma(w'_1 p'_1 + w'_2 p'_2 + w'_3 n'_3 + w'_4 n'_4) \leq \lambda \tag{5.15}$$

$$x + n_1 - p_1 = g_1 = 10 \tag{5.16}$$

$$x + n_2 - p_2 = g_2 = 7 \tag{5.17}$$

$$x + n_3 - p_3 = g_3 = 5 \tag{5.18}$$

$$x + n_4 - p_4 = g_4 = 4 \tag{5.19}$$

$$(1 - x) + n'_1 - p'_1 = g'_1 = 0 \tag{5.20}$$

$$(1 - x) + n'_2 - p'_2 = g'_2 = 0.5 \tag{5.21}$$

$$(1 - x) + n'_3 - p'_3 = g'_3 = 3 \tag{5.22}$$

$$(1 - x) + n'_4 - p'_4 = g'_4 = 4 \tag{5.23}$$

$$0 \leq x \leq 1 \tag{5.24}$$

With $\gamma = 1$, the proposer assigns equal relative importance to own desires and what the responder desires. It seems logical to take this value for the UG, as the responder has the power to reject any offer made. The result of this Chebyshev Goal Programme is a particular value for $1-x$ that minimises the objective functions of both players to the same degree $\lambda$. This value is the offer made by the proposer.

The responder is modelled in a similar way. His own criteria lead to a model as given by Eq.(5.7) - Eq.(5.12), but naturally goals and weights may differ because the proposer might actually be wrong in his ToM model about the responder's true mindset (even if assuming that they consider the same set of criteria and the same set of states). The ToM model of the responder also looks similar to Eq.(5.1)-Eq.(5.6), but again goals and weights may differ as the responder might also be wrong in assessing the true mindset of the proposer.

Any offer $1-x$ made by the proposer is now evaluated by the responder by evaluation in these models of this offer. It is accepted if $z'(1-x) \leq z(1-x)$, and rejected otherwise (with $\lambda = 1$). Mathematically, however, this is equivalent to solving for the responder a Chebychev GP based on this own versions of the model and ToM model, and finding the minimum acceptance offer $1-x'$; the offer is then accepted if $1-x \geq 1-x'$, and rejected otherwise. The distribution of the accepted and rejected offers is shown in figure 5.1.

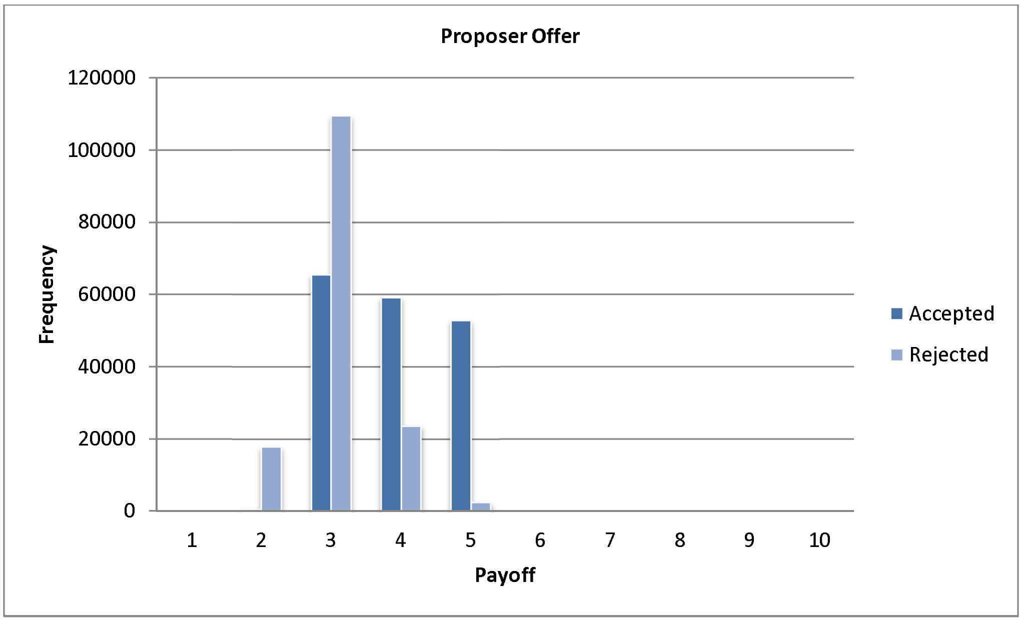


Figure 5.1: Distribution of Proposer Offers in Ultimatum Game

The Kolmogorov-Smirnov and Shapiro-Wilk tests show that the data dis-

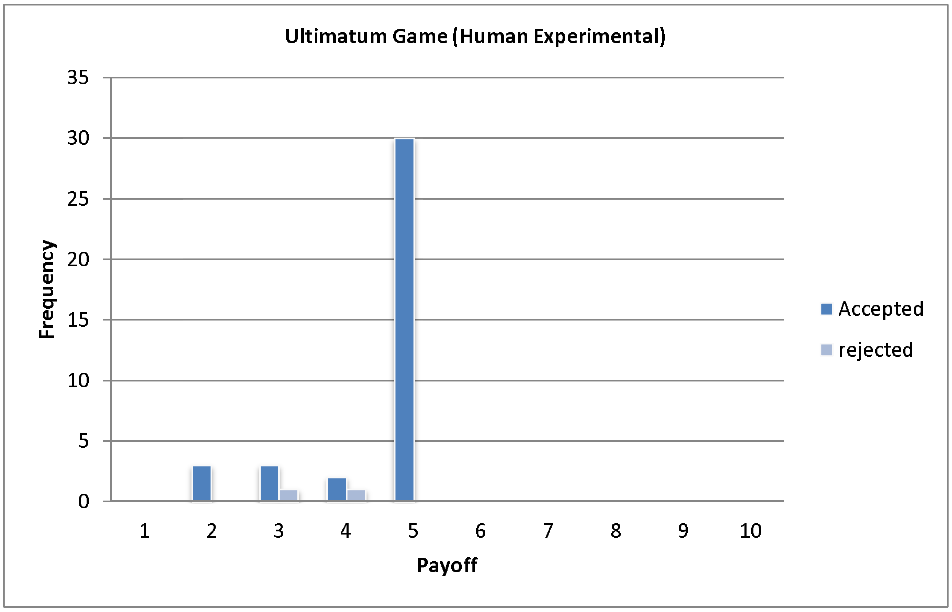


Figure 5.2: Distribution of Proposer Offers in Ultimatum Game (Human Experimental)

tribution of proposer offers appears to be non-normal, $p - value = 0.000 < 0.05$. For the statistical analysis, non-parametric tests are therefore chosen. The non-parametric Mann-Whitney test is used to show that the accepted and rejected of proposer offers are significantly different. From the Mann-Whitney test, it is observed that there is a significant difference between accepted and rejected offers at the 5% level ($U = 17472, Z = -12.096, p - value = 0.00$).

From the distribution in figure 5.1 there is a clear pattern that acceptance rate of higher offers is higher. The range of proposers' offers is between 2 to 5. This is also observed in experimental results for the UG tested with real subjects, see e.g. figure 5.2 (Haselhuhn and Mellers, 2005). The modal offer is 3 in which the rate of accepted offers, 53.73%, is higher than the rate of rejected offers, 46.27%. There is no offer lower than 2, which corresponds to the findings of Camerer (2003), suggesting that proposers do not act according to the rational notion of both players being purely selfish. The mean offer is 3.5. The mean offer in experimental studies is typically higher: 4.56 in Carter and Iron (1991), 4.16 in Prasnikar and Roth (1992), 4.67 in Forsythe et al. (1994), 4.5 in Croson (1995), 4.6 in Haselhuhn and Mellers (2005), and 4.7 in Takagishi et al. (2010).

The Ultimatum Game is quite 'stressful' for both proposer and responder due to the high uncertainty about the other player's thinking. The proposer faces uncertainty whether his offer be accepted and what kind of offer will yield the highest (expected) payoff. The responder faces ex-ante uncertainty since he does not know what the allocated sum will be. Those feelings of uncertainty, combined with the interaction with the other player, can lead to individual differences in both the value assigned to the different goals, and the weight assigned to the difference deviational variables.

### 5.3.2 Goal Criteria Distribution

In each game where proposer makes an offer to responder and it is being accepted, the goal attached with the highest weight is traced for both parties so that the information about the goal dimension of each player can be revealed. Table 5.1 shows the acceptance and rejection frequencies of proposers' offers across the criteria of goals.

Table 5.1: The acceptance and rejection frequencies of the offers across the criteria of goals.

| | **Proposer** | | **Responder** | |
|---|---|---|---|---|
| **Goal** | Accepted Offer | Rejected Offer | Accepted Offer | Rejected Offer |
| MP | 49152 | 33792 | 56192 | 26752 |
| FoR | 31424 | 51520 | 44992 | 37952 |
| CR | 43200 | 39744 | 36480 | 46464 |
| EM | 54464 | 28480 | 40576 | 42368 |

The contingency table test (A.1.5.3) was conducted to test the null hypothesis of no association between the criteria of goals and the proposer and responder decisions of accepting and rejecting the offers. From the test, it is observed that the criteria of goals and the decisions are associated to each other at 1 percent significant level ($\chi^2 = 1904.25, p - value = 0.00$). Hence, there is enough evidence to reject the null hypothesis of no association between the criteria of goals and the proposer and responder decisions of accepting and rejecting the offers. It can be concluded that the dimensions of goals; MP, FoR, CR and EM show their significant influences for proposer and responder decisions.

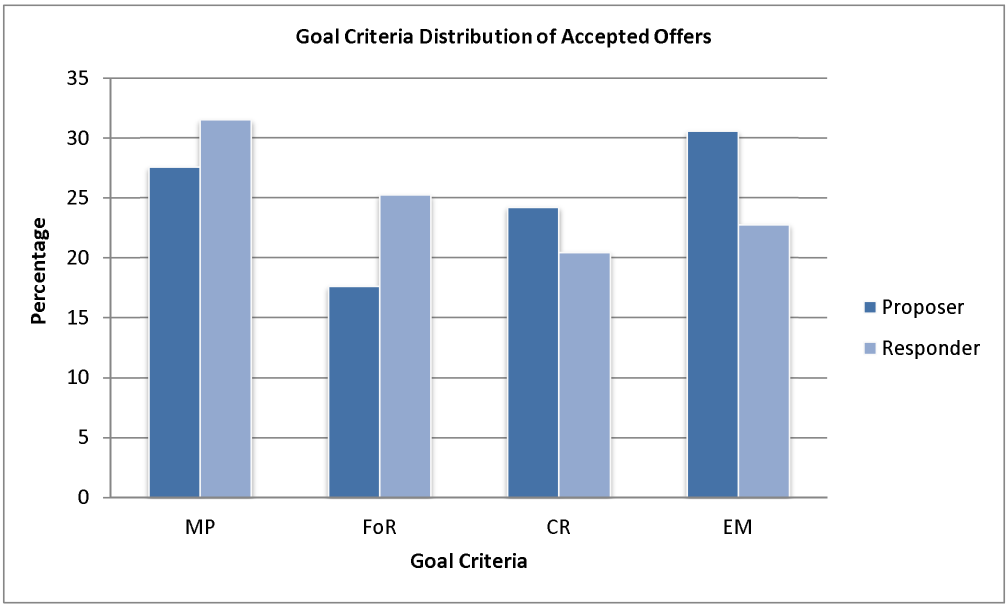


Figure 5.3: Goal Distribution of Accepted Offers

Figure 5.3 shows the distribution of accepted offers (in percentages relative to the total number of games played) for proposer and responder across the criteria of goals. It can be seen that the acceptance rate is the highest for the proposer and responder when the EM and MP goal dominate, respectively. Therefore it can be assumed that the proposer makes his 'best' acceptable offer when he is making decisions driven by emotions. For the responder, the acceptance rate is higher when making decisions on rational grounds ('Any money is good money'). The role of emotions cannot be reduced to that of shaping the reward parameters for rational choice. It seems likely that they also affect the ability to make rational choice and interact with other motivations to produce behaviour.

### 5.3.3 Sensitivity Analysis of Proposer Goal Values

A sensitivity analysis is conducted through what-if analysis in order to see the consistency of the ranked goal values chosen for the proposer. Two alternative scenarios have been considered to see the changes in the rate of accepted and rejected offers from the initial solution, *S1* in the first experiment. In the first alternative scenario, *S2*, this research would like to see the impact on the relative division of accepted and rejected offers when proposers are being more generous. For this purpose, the goal values of proposers are reduced by 1 unit for each goal in comparison to *S1*. Proposers in this alternative scenario use these new goal values, but it is important to note that in the ToM model of responders, the proposers' goal values are being kept at their original level (as in *S1*). In the second alternative scenario, *S3*, we would like to see whether the acceptance rate of offers decreases or rejection increases if proposer set more selfish goal values. For this purpose, the proposers' goal targets are increased by 1 unit from each goal from the initial solution. Again, in the ToM model of responders, the goals values for the proposer are kept at their original levels. The changes to the goal values are summarized in table 5.2.

Table 5.2: Sensitivity Analysis of Proposer Goal

| Scenario (S) | **MP** | **FoR** | **CR** | **EM** |
|---|---|---|---|---|
| **S1** | 10 | 7 | 5 | 4 |
| **S2** | 9 | 6 | 4 | 3 |
| **S3** | 10 | 8 | 6 | 5 |

#### 5.3.3.1 *S2*

Figure 5.4 shows the distribution of proposer offers in *S2*.


Figure 5.4: Distribution of Proposer Offers (S2) in Ultimatum Game

The Kolmogorov-Smirnov and Shapiro-Wilk test show that the data distribution of responder acceptance value appears non-normal, $p - value = 0.000 < 0.05$. The two-tail Mann-Whitney test (A.1.5.1) shows the rate of accepted and rejected offers is significantly different at the 5% level ($U = 15360, Z = -6.746, p - value = 0.00$).

In *S2*, the results show that most of the proposer offers are being accepted by the responder with the rejection rate is reduced to 27.33%. In comparison with *S1*, the modal offer increase from 3 to 4. The mean offer also increase to 4.1 from 3.5. These results suggest that fair offers may vary from 3 to 6 with the smallest offer is increased from 2(*S1*) to 3(*S2*).The acceptance rate is quite higher in *S2* than *S1* showing that by offer by generous proposer may help increasing the acceptance rate in UG. Note that these rates are defined as the number of games in which offers are accepted (rejected) relative to the total number of games played. It is hypothesised that the rejection rates are reduced when proposers are being more generous. The results of the hypothesis tested about these two proportions of rejection rates, 27.33%(*S2*) and 46.27%(*S1*), support the hypothesis at the 1% significance level ($Z - statistic = -159.977, p - value < 0.00$).

The figure 5.5 shows the distribution of accepted offers (in percentages) for proposer and responder across the criteria of goals. Although the acceptance rate is higher in *S2*, there is not much difference in distribution of accepted offers across the criteria of goals compared to *S1*.

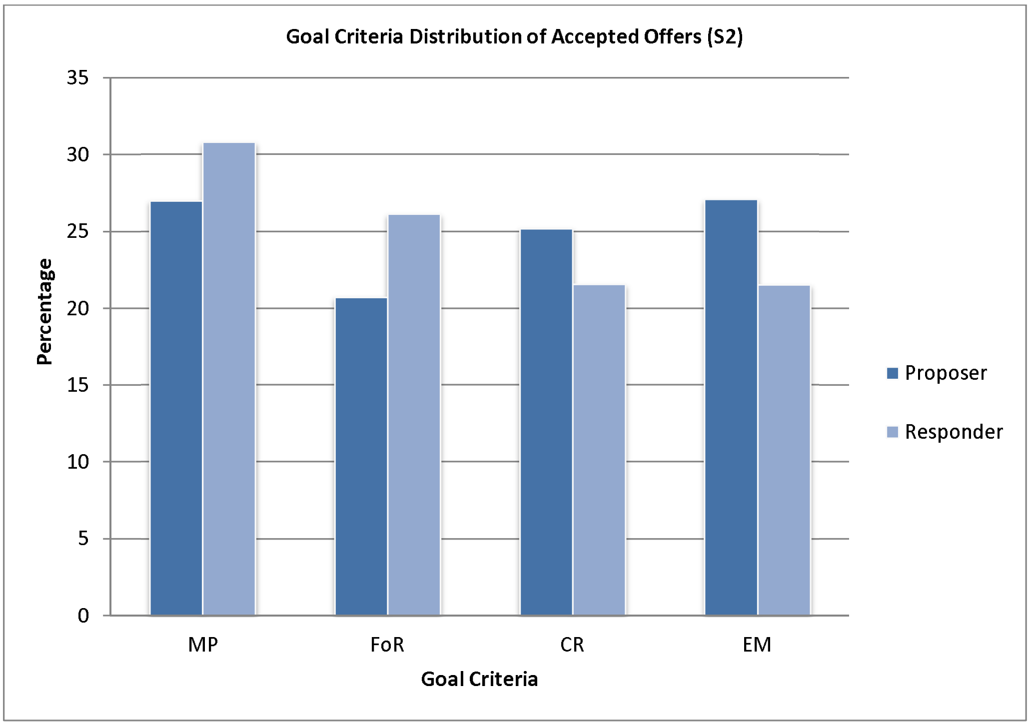


Figure 5.5: Goal Criteria Distribution of Accepted Offers (S2)

#### 5.3.3.2 *S3*

Figure 5.6 shows the distribution of proposer offers in *S3*.

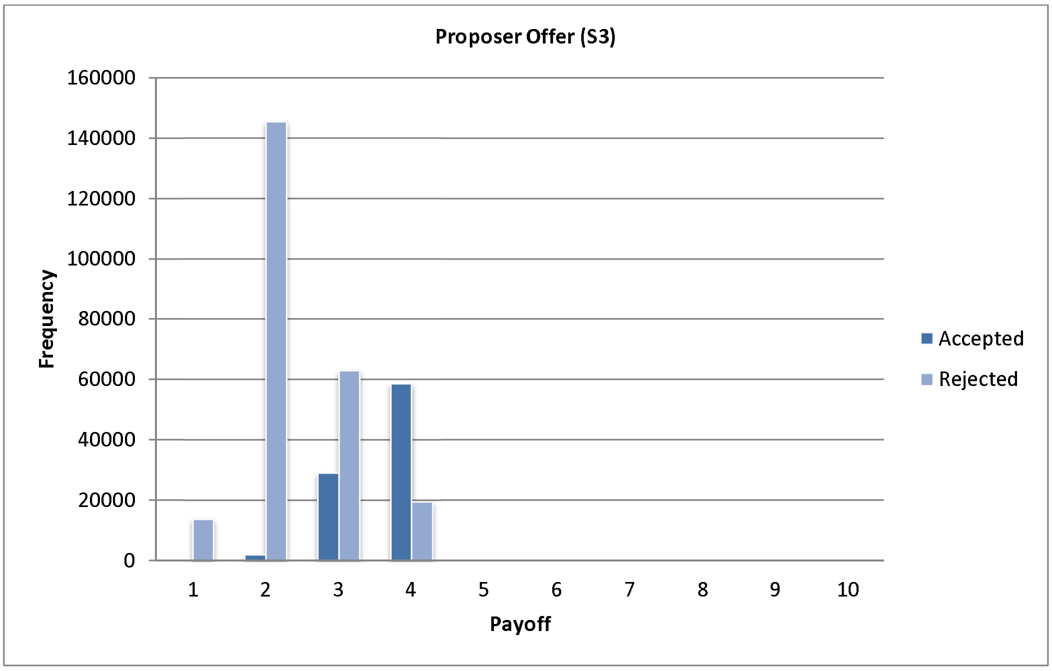


Figure 5.6: Distribution of Proposer Offers (S3) in Ultimatum Game

The Kolmogorov-Smirnov test show that the data distribution of proposer offers appear to be non-normal, $p{-}value = 0.000 < 0.05$. The two-tail Mann-Whitney test shows the rate of accepted and rejected offers is significantly different at 5% ($U = 3360, Z = -13.301, p - value = 0.00$).

The mean offer, 2.71 is quite low in comparison with *S1*. The offers vary from 1 to 4, which is also slightly lower than *S2* and *S1*. In *S3*, the results show that most of the proposer offers are less accepted by the responder with the acceptance and rejection rate are 27.06% and 72.94%, respectively.

It is then hypothesised that the rejection rates increased if a proposer being selfish when make his best offer to a responder. The hypothesis tested about these two proportions (A.1.5.2) of rejections 72.94%(*S3*) and 43.67%(*S1*), and the results support the hypothesis at 1% significance level ($Z - statistic = 221.283, p - value < 0.00$).

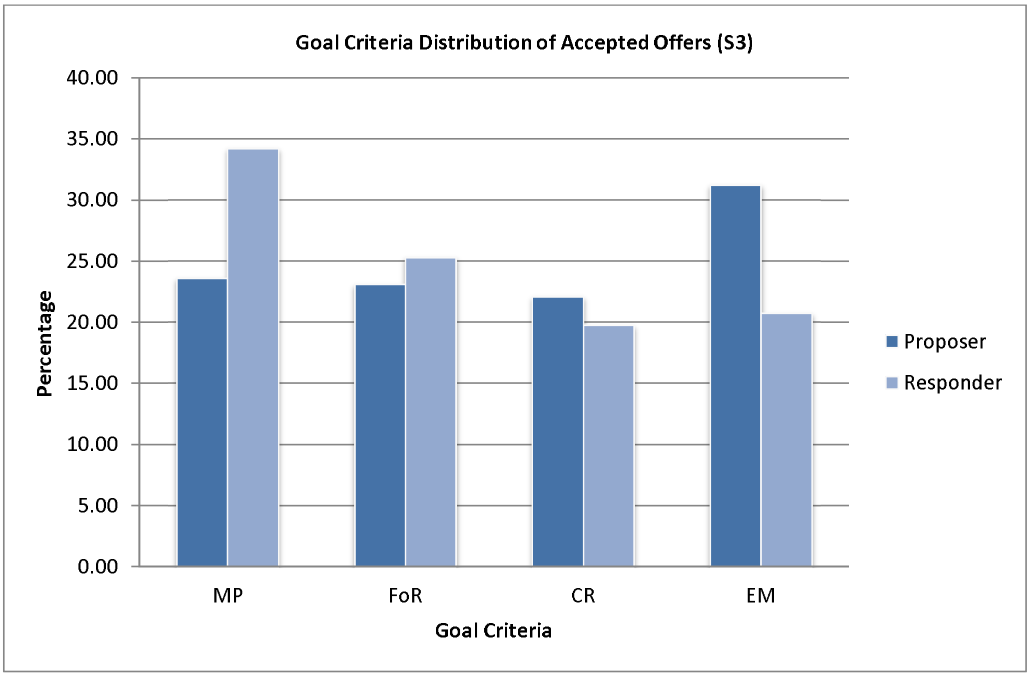


Figure 5.7: Goal Distribution of Accepted Offers (S3)

Figure 5.7 shows the distribution of accepted offers (in percentages) for each proposer and responder across the criteria of goals. Although the pattern of distribution has changed, but yet still the acceptance rate is the highest for the proposer and responder when the EM and MP goal dominated, respectively.

The summary of the acceptance and rejection rates in the three scenarios discussed in the above is represented in Table 5.3.

Table 5.3: Sensitivity Analysis of Proposer Goal

| Scenario | **MP** | **FoR** | **CR** | **EM** | Accepted Offers (%) | Rejected Offers (%) |
|---|---|---|---|---|---|---|
| **S1** | 10 | 7 | 5 | 4 | 53.73 | 46.27 |
| **S2** | 9 | 6 | 4 | 3 | 72.67 | 27.33 |
| **S3** | 10 | 8 | 6 | 5 | 27.06 | 72.94 |

As shown in study by Haselhuhn and Mellers (2005), most proposers made fair offers in the ultimatum game, but the reasons behind those fair offers appeared to vary. This study has shown that the variety in fair solution exist for the reason that fairness in human decision making is asymmetry according to different dimension that one player carries when making decision. In fact, the decision sometimes is not rational but fairly enough for that particular player.

#### 5.3.3.3 Total Profits by Proposer and Responder

Recall that if the responder accepts the proposer's offer, then both of them receive a payoff. Otherwise, they will get nothing out of it. We are hence interested in finding out how much of the total amount of money available (the £10 times the total number of games) will find its way to proposers and responders. From a 'social welfare' point of view, we would find it arguably best when all this money would be awarded. However, the cost of having two players that need to divide it, and have their own idea about fairness, is that not all of this money will be awarded. Furthermore, their judgements about fairness influence how the money is distributed amongst them.

This section analyzes the monetary profit each proposer and responder gain if they win the game in the three different scenarios, *S1*, *S2* and *S3*. Table 5.4 shows the total profits obtained by both proposer and responder in each scenario. Remark that the total profit obtained, defined as the total amount collected in games with offers accepted relative to the total amount available across all games played, equals that of the total acceptance rate found in the previous section. The social welfare is increased in scenario *S2* relative to *S1*. It is hence better for social welfare that proposers are more generous than what responders think in their ToM about proposers. In addition, this also increases the wealth of both proposers and responders. The distribution of proposers and responders profit in percentage in each scenario shown in Figure 5.8, 5.9 and 5.10. From the distributions, it can be seen the proposers and responders profit are more equally distributed in *S1* in comparison to *S2* and *S3*.

Table 5.4: Percentages of Profit by Proposers and Responders

| Scenario | **Proposer** | **Responder** | **Total Profit** |
|---|---|---|---|
| ***S1*** | 32.65% | 21.07% | 53.72% |
| ***S2*** | 41.67% | 31.00% | 72.67% |
| ***S3*** | 17.23% | 9.83% | 27.06% |

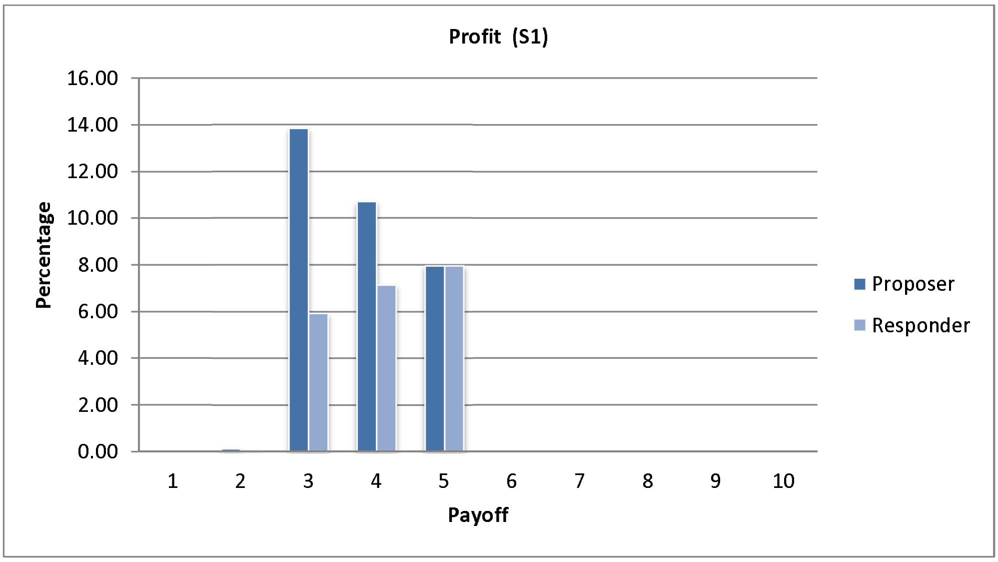


Figure 5.8: Distribution of Profit by Proposers and Responders (S1)

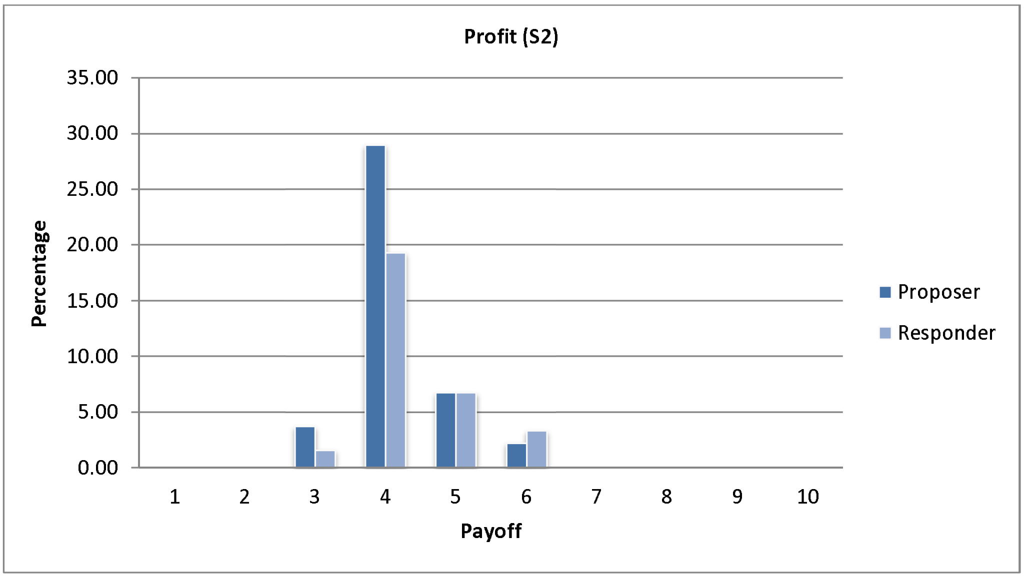


Figure 5.9: Distribution of Profit by Proposers and Responders (S2)

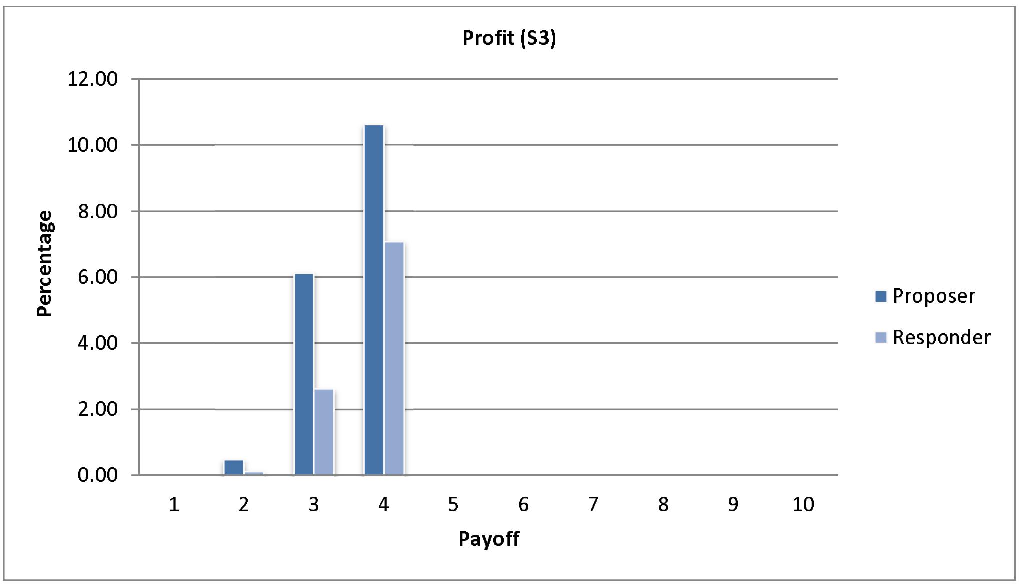


Figure 5.10: Distribution of Profit by Proposers and Responders (S3)

### 5.3.4 Sensitivity Analysis of Responder Goal Values

A similar sensitivity analysis is conducted through what-if analysis in order to see the consistency of the ranked goal values chosen for responder. Two scenarios have been considered to see the changes in the rate of accepted and rejected offers from the initial solution, *S1* in the first experiment. In the first alternative scenario, *S2*, responders are being more generous in terms of accepting offers. The goal values of proposers are reduced by 1 unit for each goal relative to what they are in *S1*. Responders behave, in effect, more rational. Note that in the ToM of the proposers, the goal values of responders are kept at their original levels. In the second alternative scenario, *S3*, responders place a higher emphasis on fairness. For this purpose, the responders' goal values are increased by 1 unit from each goal from the initial solution. Responders are hence placing less emphasis on monetary payoff or the need to acquire it, but place more emphasis on achieving reputation and may react with more righteous anger to unfair offers. Again, proposers do keep using the original values in the ToM model of responders. The summary of the acceptance and rejection rates in the three scenarios is given in Table 5.5.

Table 5.5: Sensitivity Analysis of Responder Goal

| Scenario (S) | **MP** | **FoR** | **CR** | **EM** | Accepted Offers (%) | Rejected Offers (%) |
|---|---|---|---|---|---|---|
| **S1** | 0 | 0.5 | 3 | 4 | 53.73 | 46.27 |
| **S2** | 0 | 0.3 | 2 | 3 | 50.91 | 49.09 |
| **S3** | 1 | 1.5 | 4 | 5 | 41.42 | 58.58 |

#### 5.3.4.1 Discussion of results

In the first alternative scenario, proposers make offers under the belief that responders are as emotional as in the base case scenario. However, responders are in fact more rational. We would expect that acceptance rates would hence be higher, but this is contradicted by the results of the model. To explain this counter-intuitive result, we should compare with Figure 5.1. At lower offers, the responder feels that the proposer is doing him- or herself short on the dimensions of reputation and emotions. Hence, the responder rejects offers in the range between 3 and 4 more often than in the baseline scenario. These offers are not low enough for the responder to start reacting emotionally in terms of feeling righteous anger. The responder, having a ToM about the proposer in which he or she is truly concerned about reputation and fairness, feels that offers in this range 3 to 4 are too low for the proposer to score high on these goals. This also explains why proposers, when being more emotional and offering higher values, will see their acceptance rates relatively increased. It is an interesting phenomenon in the model, and the question to which degree this can truly correspond to reality is left for further research.

In the second alternative scenario, proposers make again the same offers as in the base case scenario. However, responders react more emotionally to offers, so acceptance rates should decline. This is confirmed by the model's experiments.

#### 5.3.4.2 Total Profits by Proposer and Responder

The power of the responders to influence the division of payoff cannot be denied. In the ultimatum game, the responder is primarily reacting to the behaviour of the proposer. Table 5.6 shows the total profits obtained by proposer and responder in each scenario. It can be seen from the table, the social welfare is decreased in *S3* in comparison to *S1* and *S2*. It is, hence, showing responders being more 'selfish' or too demanding than what proposers think in their ToM model which in the end would create losses to the society.

Table 5.6: Percentages of Profit by Proposers and Responders

| Scenario | **Proposer** | **Responder** | **Total Profit** |
|---|---|---|---|
| ***S1*** | 32.65% | 21.07% | 53.72% |
| ***S2*** | 30.62% | 20.28% | 50.90% |
| ***S3*** | 24.64% | 16.77% | 41.41% |

## 5.4 Dictator Game

In this version of the game, the responder cannot reject the offer and there is thus no longer the need to have a responder's model. The proposer's model is simplified as certain criteria do no longer play a role. Indeed, there is no longer the fear of rejection. It is thus within reason to take $w_2 = 0$. Although the proposer might still have a ToM of the responder, she knows the responder is powerless, and thus it is reasonable to take $\lambda = 0$ in her Chebychev GP model. Concern about reputation, because it is in reference to an outside peer group, will still play a role. This can reflected in the model by considering a range of values for $w_3 > 0$. Feelings of sympathy may still be present, but for simplicity, we have set $w_4 = 0$ (taking non-zero values produces similar results).

The result is a simple model in which monetary pleasure is balanced with concern about reputation. Keeping goal values as in the UG, 21 experiments are conducted on computer using different relative weights for the two criteria in steps of 0.05 (keeping their sum always equal to 1 for normalisation). An additional 21 experiments are then performed by setting the target for concern about reputation to 6 instead of 5, based on the argument that in the DG the peer group may no longer expect that an equal split is what dictators need to respect. The DG model is then solved by using Lingo Programming (A.1.4). Results of these 42 games are summarised in Figure 5.11. The average offer in DG is 2.36, and the distribution is multi-modal, with a large peak at low offers and another large peak close to the 0.4 to 0.5 range. While real experiments show a wider range of offers in between 1 and 5 (Haselhuhn and Mellers, 2005), the bi-modal character of the distribution from these experiments with the GP model do reflect the results from experiments with real subjects reported in the literature (refer figure 5.12).

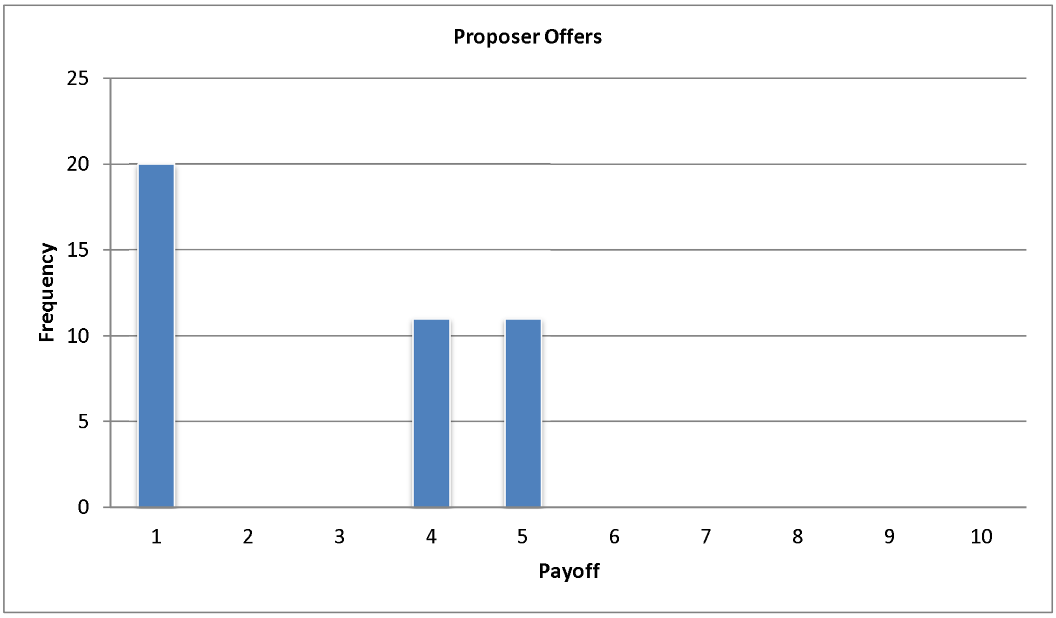


Figure 5.11: Dictator Game

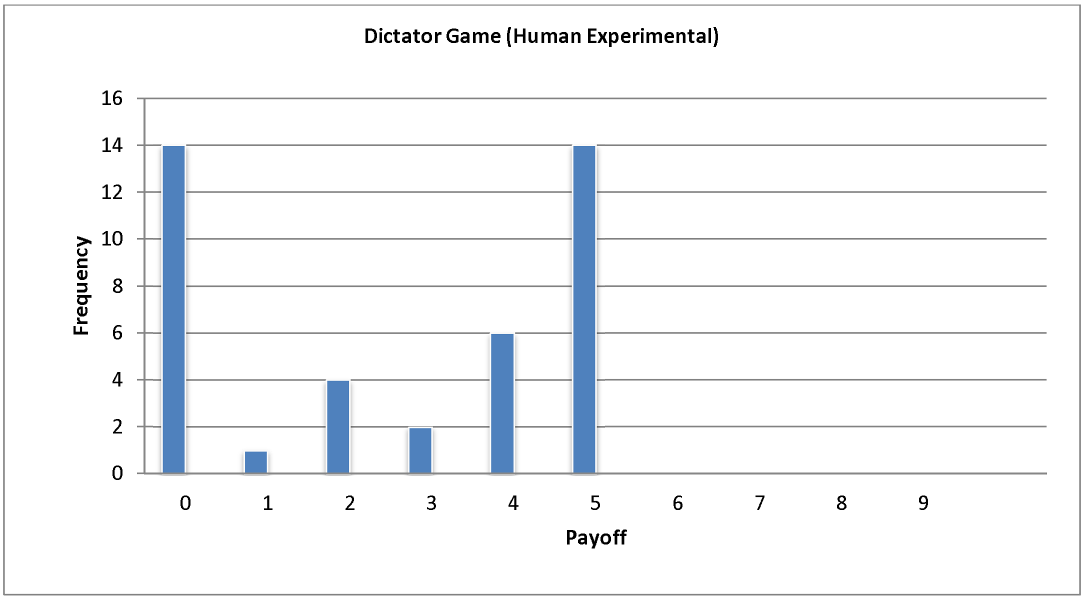


Figure 5.12: Distribution of Proposer Offers in Dictator Game (Human Experimental)

## 5.5 Double-Blind Dictator Game

In the DBDG, the proposers and responders are told by the organisers of the game that they will not know from each other who offers what to whom. However, even while dictators are told they will not be unmasked as being greedy, they might not believe fully in the honesty of the organisers, and thus still have some fear that that the organisers of the game will reveal who made the lowest offers. This aspect of the DBDG may be encountered in real life situations where, if subjects think they can get away with being greedy, they will be very tempted to do so. However, some fear of being unmasked at some later point in time might still exist, even if the chances are low.

It is thus assumed that the DBDG is a simplification of the DG model, with relatively large weight for the first criterion ($w_1$ is large), and a smaller but non-zero weight for the third criterion ($w_3 > 0$). All other criteria will carry zero weight, as in the DG.

In the computer experiments, the weights combinations for $w_1$ and $w_3$ are again chosen in steps of 0.05 but within the more limited range defined by a minimum weight for monetary pleasure set at 0.75 and a maximum weight for reputation set at 0.25. The results are not shown on a graph in this case, as all offers made are 0 (or the minimum offer required). Despite carrying some weight, concern about reputation is not of sufficient importance to influence the result. This again corresponds reasonably well with experimental findings from games with real subjects in the literature.

## 5.6 Nash Equilibria

Nash Equilibrium is a term used in game theory to describe the equilibrium where each player's strategy is optimal given the strategies of all other players. In the UG, there are said to be many possible (weakly dominated) Nash equilibria, each of which corresponds to an offer in which the proposer keeps the value $x = x'$, where $1 - x'$ is the minimum value that the particular responder still accepts (Roth and Erev, 1995). The Chebyshev GP models in this paper calculate such Nash Equilibria, with the understanding that, in the context of uncertainty about the other player's mindset, both proposer and responder each calculate their own Nash equilibrium based on the ToM of their opponent. If both players have an accurate ToM of the other player and assign equal importance to it, these Nash equilibria coincide and thus offers are made that are always accepted. If one or both of them are wrong in their ToM model or differ in the way it is considered of importance, the offer could be accepted or rejected.

When playing the UG repeatedly, one would assume that both types of players would learn, thereby possibly updating their own model as well as their ToM, and may arrive in the end at a long-term equilibrium. The subgame-perfect equilibrium for the UG is thought to be that the proposer offers the minimum amount to the responder, but it is well-known that this equilibrium is not reached in practice, even after repeatedly playing for a long time. Roth and Erev (1995) and Binmore (2007) found that the first few rounds are crucial to explain the dynamics and Nash Equilibrium the game tends towards. That this is far from the subgame perfect equilibrium is explained by Roth and Erev (1995) from the fact that the proposer quickly (or initially) realises that she has to lose a lot from making a low offer (a high $x$), while a responder loses less by rejecting a low offer. These initial conditions are enforced in our GP models since several goals are all driving offers and acceptance cut-off points to higher values, in particular: the proposer's dimension of goals and the adherence to social norms (arguably close to a 50/50 split in the UG.

It is believed that the four criteria of goals of the player offer give some flexibility to model and predict outcomes of other variations of the UG. For example, in the modified UG in which the responder would gain $1 + x$ on acceptance of $1 - x$ by the responder, the perfect equilibrium is still $x \approx 1$. However, the effect in the GP model depends on whether the information that proposers gain an extra 1 is made known to responder or not. In the former case, not only is the fear of rejection for proposer more clearly present, the level at which righteous anger starts to play will now kick in closer to what both players realise to be the new social norm $x \approx 0$. Offers made have to be much more generous and close to the full offer. If the information is not known to responder, however, proposers should realise they may still play according to the old social norm, and with the same level of their own fear for rejection. The outcome would thus have to be the same as in the normal UG. However, as the proposer's fear of rejection does (subconsciously) increase somewhat, as well as perhaps his target of the social norm, the GP model would predict that offers made would, on average, be slightly higher.

## 5.7 Placing the UG in real-life situations

Confronted with the UG, a player is likely to use the type of reasoning that helps in real-life UG-like situations. An example is presented that may provide further support for the GP model presented in this research. Consider the way a trades person sets his price for a particular job for a household, and how the household decides to accept or reject the offer. When a plumber, for example, is called for by a household in need of a repair, he will tend to increase his price if his current workload is high, and decrease it if it is low (fear of rejection). He also considers not deviating too much from the going rates of other plumbers of similar reputation in the neighborhood (concern about reputation). If he has conducted work for this household before, he might adjust his price to the level he has been charging in the past to this household (trust), or his price may be somewhat lowered the more he wishes to do more work for this household in the future (sympathy). At the same time, the plumber builds a ToM model of the household's situation. The more urgent the repair, and the more the scarcity of available plumbers in the neighborhood, the higher the price the household should be willing to pay (fear of rejection). The general financial status of the household, derived from observable facts as e.g. the neighborhood and type of house or car they have, may influence the price range that he considers (concern about reputation). Depending on his assessment of the household's knowledge about the plumbing business, the plumber assesses how severely too high prices above the going rate will be detected by the household (righteous anger). The household's model is a mirror image of the above description, but based on its own assessment of the situation. The plumber might actually try to influence this assessment by giving (mis)information about his fear of rejection, reputation, and sympathy. Psychology and neuroscience indicate that not all these factors might (need to) be consciously considered, but that the collective of guiding signals driving decisions will in part be composed of subconsciously ruled goal seeking processes.

Another example is selling and buying a house. In many countries, buyers typically make offers that deviate from the seller's official asking price, and the questions are how the buyer arrives at his price offer, and whether the seller accepts or rejects this particular offer. We believe that the UG model could be a valid starting point of developing a decision model to analyse these UG-like but real-life decision problems. One of the interesting aspects to investigate is the effect of signalling, or the information that an intermediate person (the estate agents) reveals of the other party in order to facilitate the successrate. The latter aspect might also call for modeling estate agents and their individual desires explicitly.

Another example is how the government can understand what stimulates people on benefits to accept employment, or criminals to better their lives. Important lessons for the UG models here developed is that it will be hard to convince people to start working or say no to crime if they do not respect the government's opinions but rather the values they share with their peer group.

## 5.8 Concluding Remarks

In this chapter, a GP framework is proposed in which asymmetrical fairness is viewed as a tradeoff between players to reach decision in UG, DG and DBDG game. It has shown that a large body of experimental evidence may be explained by a relatively Chebychev GP in which players experience four different goal dimensions (MP, FoR, CR and EM) responses depending on their own and opponent's desire goals. The Chebychev GP model then be able to suggest solutions that balances all these dimension for each player in order to produce fair solution.

Rejected offers in this approach can only occur when players differ in their ToM of each other, or when the value of $\lambda$ differs from 1. The latter parameter measures the relative importance a player assigns to the desires of the other player. For the UG, it naturally should be (close to) 1, while for DG and DBDG it should be (close to) 0. An open question is whether players (in the UG) try to build an accurate ToM of the other player, or a personal idealised version how they think they other player should behave. While the first would be arguably the best strategy for playing the UG, the many disputes about what is a fair solution to real coordination problems (e.g. local neighborhood disputes, taxes and benefits, environmental issues) indicate that the latter is perhaps more often true.

A feature of efficient biological computation is that our brain reduces decision processes to become automated as much as possible. This process of automation has been constructed under typical decision making situations in our distant past, and is thus optimised to handle repeated occurrences of coordination in social groups. It is likely the most important reason why subjects consider, to some degree, criteria that are not considered rational from within the precise rules of the single-shot UG. Based on the valuable information derived from real experiments in the fields of evolutionary psychology and neuroeconomics, it turns out that righteous anger and fear of rejection, concerns about reputation and peer group's expectations, and even aspects of sympathy and trust may thus influence decisions. It has been proved through contingency table test that there is enough evidence to reject the null hypothesis of no association between the types of goals and the proposer and responder decision of accepting and rejecting the offers. We have modeled these games using Goal Programming, including these features, and distinguishing between the players' model of themselves and a model they construct of the other players. This allowed the formulation of fairness as a Chebychev GP. Both players thus consider the assumed desires of the other player to decide what would be a fair offer.

These games are discussed from the perspective of evolutionary psychology. Perhaps the most important feature in the UG is that responders are driven by a sense of *righteous anger* and *punish* selfish proposers; proposers anticipate this and, having fear of rejection, make offers just generous enough to be accepted. Because this fear of rejection is no longer present in the DG, offers become much smaller. There could also be a *concern about reputation* for proposers. This would explain that the offer in the DG still tends to be larger than the minimum offer, because the proposer worries about getting a bad reputation of being selfish that could come back on him *in the long run.* This feature can be interpreted as having a natural desire to fit to what is expected good behaviour from the social peer group that the proposer feels connected to. Although it shouldn't play any role in a single-shot game played only once, evolutionary psychology explains why we tend to think in this way, since the peer group holds possibilities of playing these games in the future, and we tend not to want to jeopardise our future successes. Others have argued that this behaviour is just the result of proposers having a natural tendency towards fairness. The latter explanation, however, cannot explain why in the DBDG most proposers do keep almost everything for themselves.

# 6 Fairness Model with Pooling Formulation

## 6.1 Introduction

In this chapter, the interest is to develop a model of human decision making process for cooperation using a weighted GP approach which involves more than two players. In this model, this chapter introduces fairness into models of cooperative games to seek answers to the following questions: (1) which coalition is likely to form; and (2) how are the payoffs distributed amongst its members? This approach is called the 'pooling formulation' as it is based on a notion of fairness that a party derives from its option to join one amongst all potential subcoalitions or pools (including the grand coalition).

The approach differs from a traditional multiobjective model in that goals related to each of the decision makers' individual preferences are explicitly introduced. This research consider that each player has a goal with respect to his own monetary payoff, and a goal towards assuring a balance between his own payoffs and payoffs that other members of a coalition receive. The first is called the profit goal whilst the latter is the fairness goal. Players are allowed to differ in their beliefs what a fair payoff distribution is. It will be through numerical examples, the importance of players' sharing common beliefs about fairness with respect to the formation of a coalition and its payoff distribution will be further examined. The fairness GP model is then applied to Drug and Land game for numerical example.

## 6.2 Fairness GP Modeling

When a group of decision makers interact with each other in a project on cooperation, they are in general faced with conflicting objectives as each is in principle interested in maximising their own payoff. In addition, however, each player also realises that the power of any subcoalition is not based on its total worth, in contrast to what is expressed in classic cooperative game theory that was reviewed in Chapter 4, but rather that this depends on the ability of this subcoalition to find solutions that find agreement among its members about the degree of equity of the distribution of the payoffs among its members.

Let $N$ be the set of $n$ players $N = \{1, 2, ..., i, ..., n\}$ that are able to form $m$ pools from the set $M$, $M = \{S_1, S_2, ..., S_j, ..., S_m\}$. Each player considers a goal with respect to profits and a goal with respect to fairness about the distribution of payoffs in any potential pool it can participate.

For the profit goal, each player $i \in N$ will set an aspiration value as their profit target $g_i^p$, where $p$ refers to profits. Let $X_i$ measure the payoff that player $i$ will eventually receive, $\forall i \in N$, then the profit goal function expressed as in equation 6.2. In this goal function of a player $i$, the negative deviation, $n_i^p$ is minimised from his aspiration value.

In addition, each player considers a fairness goal that we assume is of similar form as in Fehr and Schmidt (1999)'s inequity aversion model. Let $S^i$ be the set of pools in which $i$ can participate, and $S^i \setminus \{i\}$ the set of such pools excluding the trivial pool $\{i\}$. Let $X_{iS}$ be the payoff that player $i$ can receive in pool $S \subset S^i$, $\forall i \in N$. For any pool $S \in S^i \setminus \{i\}$, there are players $k \in S$, $k \neq i$ that receive payoff $X_{kS}$. Fairness of any pool solution to the cooperation game then means for player $i$ the ability to meet a target value $g_i^f$, where $f$ refers to fairness. The fairness goal function expressed as in equation 6.2.

In this goal function, both negative and positive deviation, $n_i^f$ and $p_i^f$ are minimised from the aspiration value to express their fairness concern.

In this model, players can differ not only in the target values for profit and fairness goals, but also, in the relative weights they associate to deviational variables they want to minimised in goal functions. For the profit goal, a particular weight, $\alpha_i^p$ is attached to the negative deviation, $n_i^p$ in the objective function to reflect player's concern over his profit target. While for fairness goal, the particular weights, $\alpha_i^f$ and $\beta_i^f$ are attached to the negative and positive deviation, $n_i^f$ and $p_i^f$ respectively, to reflect player's concern over disadvantageous inequity and advantageous inequality. The parameter $\alpha_i^f$ represents a player's dislike towards having a payoff less than the average across all other players in any potential pool, and $\beta_i^f$ measures how much the player dislikes having more than the average of all others in any potential pool. According to Fehr and Schmidt (1999), the parameter $\beta_i^f$ should be less than or equal to parameter $\alpha_i^f$, as people tend to be more concerned when their payoff is less than the average amongst other players.

All these parameters are then further normalised by making the total sum of parameter $\alpha_i^p$, $\alpha_i^f$ and $\beta_i^f$ equal to 1 in the objective function, $z$ and this expressed as in equation 6.1

The above goals are subject to the set of core and pool constraints. For the core constraint, as the fairness model is based on individuals and their perception of fairness in possible pools, we do not incorporate the typical subgroup rationality conditions that would restrict solutions to be in the core. Hence, the subgroup rationality treated as soft constraints in this model. For any pool that could form, however, we do have to include a constraint that its members cannot receive more than the total worth of this pool. In addition, it does seem logical to restrict the payoff of each player should get is at least as much as when that player do not join any coalition. These constraints use the classic notion of the characteristic function value $v(S)$, where $S \in M$.

Let $S_j$ be a decision variable that takes the value 1 if the pool is selected, and 0 otherwise, $\forall S_j \in M$. The general algebraic formulation of the fairness model is then as follows:

$$Min\ z = \sum_{i=1}^{n}((\alpha_i^p n_i^p) + (\alpha_i^f n_i^f + \beta_i^f p_i^f)) \tag{6.1}$$

*Profit goals*

$$X_i + n_i^p - p_i^p = g_i^p, \forall i \in N \tag{6.2}$$

*Fairness goals*

$$\sum_{S \in S^i \setminus i} [X_{iS} - \frac{1}{|S|-1} \sum_{k \in S, k \neq j} X_{kj}] + n_i^f - p_i^f = g_i^f, \forall i \in N \tag{6.3}$$

$$X_i \leq \sum_{S \in S^i} X_{iS}, i \in N \tag{6.4}$$

*Core constraints*

$$X_i \geq V(i), i \in N \tag{6.5}$$

$$\sum_{i \in S} X_i + n_S - p_S = V(S), \forall S \subseteq N \tag{6.6}$$

*Pool constraints*

$$\sum_{i \in S} X_{iS} \leq V(S)S_j, \forall S \in M \tag{6.7}$$

$$\sum_{S \in S^i} X_{ij} \leq V(S)S_j, \forall S \subseteq N \tag{6.8}$$

$$S_j \in 0, 1, \forall S \subseteq N \tag{6.9}$$

## 6.3 Asymmetrical Fairness Preferences

Although decision makers' preference over each objective compared to another would give a better picture of the weights, they can be judged by assigning differential weights from both the profit and fairness viewpoints. There is also evidence that some proposers have a preference for fairness and willing to give up money in order to produce equal payoff (Forsythe et al., 1994). From the behaviour of the players in experimental studies observed in the literature, three different preferences are retrieved by assigning a diverse set of weights in each preferences. The procedure begins with comparing all the alternatives with respect to preemptive weights attached to each goal.

Each player is represented by a strategy specifying how the player behaves when it interacts with the other players. We base ourselves on Dannenberg et al. (2007) to classify the players' individual attitudes and retrieve associated values for their $\alpha$ and $\beta$ parameters in the model. By assuming this, we actually assume that the players behave consistently with these values throughout, and keep their preference parameters fixed. However, further the sensitivity analysis is being conducted to parameters to see whether it has significant changes in the outcome.

Table 6.1: Asymmetrical Preferences

| Parameter | $\alpha_i^p$ | $\alpha_i^f$ | $\beta_i^f$ |
|---|---|---|---|
| Selfish **(S)** | 1 | 0 | 0 |
| Very little inequity averse **(DHI)** | 0.7 | 0.2 | 0.1 |
| Fair **(F)** | 0.5 | 0.35 | 0.15 |
| Highly inequity averse **(HI)** | 0.1 | 0.6 | 0.3 |

We expect that individualists are more focused on maximizing their own payoff, and will be less impressed with outcomes that maximize group payoffs and share this equally. From table 6.1, a selfish player will put all the weights on his profit goal and does not care at all about his fairness goal. It shows that people with individualistic orientation pursue to maximize their own payoff with no regards for the outcomes of others. The second type of player is slightly inequity averse by still placing more weight on his profit goal and some concern about fairness. We assume that a fair player should put both goals as equally important. Finally, a highly inequity averse player will put more concern on their fairness goal and much less on their individual profit goal. This kind of player will strive for equality even if this means giving up his own payoffs. The latter three orientations are primarily concerned with maximising the payoffs for both self and others, but to different degrees.

This research believe that to make cooperation successful, one of the most important objective is to strengthen the group ties and increase people's identification within the group, so that members become motivated and do not leave the coalition. An increased group identification may reduce the psychological distance between the members in the group so that they perceive each other as similar in terms of their aspiration goals. By identifying each player preferences over fairness, it may hope the form of any coalition or subcoalition will be successful cooperation and stable in the sense of fairness focusing on individual rather group maximising.

## 6.4 Application of Fairness Model in Games

### 6.4.1 Drug Game

In the application of the fairness model, the simulation experiments were conducted in which players differ in their preferences which can be distinguished into a $(\boldsymbol{S})$, $(\boldsymbol{DHI})$, $(\boldsymbol{F})$ and $(\boldsymbol{HI})$ player as displayed in table 6.1. For the experiments which considering every player has the same preferences, the results obtained show the alternative solutions exist. Next, the experiments in which players differ in their preferences as in table 6.2 were conducted by considering every possibility of the preferences. There are 12 games to be played to see which coalition will be formed and how payoffs be allocated among players. The drug game model is then solved by using Lingo programming (A.2.1).

Table 6.2: Experiments on Asymmetrical Preferences

| **Player** | **Game** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| N | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | S | S | S | DHI | DHI | DHI | F | F | F | HI | HI | HI |
| 2 | F | F | HI | HI | HI | F | HI | HI | DHI | F | F | DHI |
| 3 | HI | DHI | DHI | F | S | S | DHI | S | S | DHI | S | S |

In the drug game, introduced in Chapter 3, no player can receive any profit if not joining a pool with at least one other member. Player 1 has some authority in this game as player 1 has the power to decide to either join up with player 2 or 3. To test validity of the model, first, only profit goal, $g_i^p$ has been considered in the drug game. In this case, it is assumed that every player's desire is to reach the maximum target $g_i^p$. It has shown the results obtained are the same as what suggested by the core solutions $\{X_1, X_2, X_3\} = \{1000000, 0, 0\}$, where all the profits go to player 1. Next the fairness goal, $g_i^f$ being considered in the model in order to induce more fairness payoffs allocation between coalition members. For this reason, the series of experiment are conducted to observe the effects of asymmetrical of fairness in coalition formation between players. For $g_i^p$, it is natural to assume that every player will aim the maximum target of profit which is 1000000 (reflects self interest) and for $g_i^f$, every player will try to reach the equality target (reflects norms of fairness). By keeping the goals constant, the asymmetrical of fairness between players are represented by the weight values attached to the deviation variables of $g_i^f$ as can be seen in Table 6.1. The results shown in Table 6.3.

Table 6.3: Drug Game

| **Game** | **Player** | | | **Result** | |
|---|---|---|---|---|---|
| N | 1 | 2 | 3 | Coalition | Payoff |
| 1 | S | F | HI | {S,F} | {500000,500000} |
| 2 | S | F | DHI | {S,DHI} | {500000,500000} |
| 3 | S | HI | DHI | {S,DHI} | {500000,500000} |

Table 6.3: Drug Game cont'd

| **Game** | **Player** | | | **Result** | |
|---|---|---|---|---|---|
| N | 1 | 2 | 3 | Coalition | Payoff |
| 4 | DHI | HI | F | {DHI,F} | {500000,500000} |
| 5 | DHI | HI | S | {DHI,S} | {500000,500000} |
| 6 | DHI | F | S | {DHI,S} | {500000,500000} |
| 7 | F | HI | DHI | {F,DHI} | {500000,500000} |
| 8 | F | HI | S | {F,S} | {500000,500000} |
| 9 | F | DHI | S | {F,S} | {500000,500000} |
| 10 | HI | F | DHI | {HI,DHI} | {500000,500000} |
| 11 | HI | F | S | {HI,S} | {500000,500000} |
| 12 | HI | DHI | S | {HI,S} | {500000,500000} |

As can be seen in Table 6.3, coalition with a ***HI*** player is never be an option. Whilst the ***HI*** player prefers to join a coaltion with a ***S*** player. However, the ***S*** player prefers to join a coalition with a ***DHI*** player rather than ***F*** player. This result is consistent when the ***DHI*** player also prefers to join a coalition with the ***S*** player rather than ***F*** player. For the ***F*** player, coalition with the ***S*** player is a preference. This cooperation preferences can be illustrated in figure 6.1.

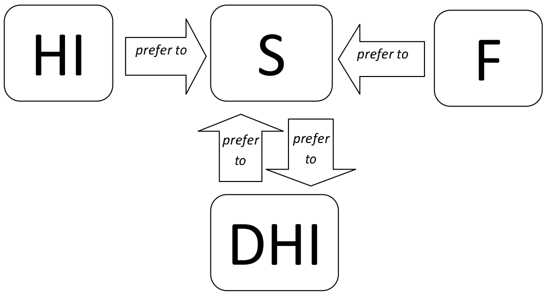


Figure 6.1: Asymmetric Preferences in Drug Game

The systematic what-if analysis of the GP model was carried out for each coalition to see whether there is any changes that will change the exist coalition and payoff allocation between players as obtained in table 6.3. First the parameter value of $g_i^p$ is being reduced per unit to see its effect on coalition formation. If there is no changes, the parameter value of $g_i^f$ was investigated to find the satisfice yet feasible break point that will change the exist coalition formation and its payoff allocation. The summary of this analysis is shown in table 6.4. Case I represent the coalition formation from the various preemptive perceptions of players towards both goals, $g_i^p$ and $g_i^f$. Whilst Case II represent the new coalition formation with the minimum changes in parameter value of $g_i^p$ and $g_i^f$ that will change the coalition formation.

Table 6.4: Drug Game: Sensitivity Analysis

| Game | Case I(*) | | Case II(**) | |
|---|---|---|---|---|
| N | Coalition | Payoff | Coalition | Payoff |
| 1 | {S,F} | {500000,500000} | {S,HI} | {750000,250000} ($g_1^f = 500000$, $g_3^f = -500000$) |
| 2 | {S,DHI} | {500000,500000} | {S,F} | {750000,250000} ($g_2^f = -500000$) |
| 3 | {S,DHI} | {500000,500000} | {S,HI} | {850000,150000} ($g_2^f = -700000$) |
| 4 | {DHI,F} | {500000,500000} | {DHI,HI} | {750000,250000} ($g_1^f = 400000$, $g_2^f = -500000$) |
| 5 | {DHI,S} | {500000,500000} | {DHI,S} | {950000,50000} ($g_1^f = 900000$, $g_2^f = -500000$) |
| 6 | {DHI,S} | {500000,500000} | {DHI,S} | {950000,50000} ($g_1^f = 900000$, $g_2^f = -900000$) |
| 7 | {F,DHI} | {500000,500000} | {F,HI} | {950000,50000} ($g_1^f = 600000$, $g_2^f = -900000$) |
| 8 | {F,S} | {500000,500000} | {F,S} | {950000,50000} ($g_1^f = 900000$, $g_2^f = -900000$) |

Table 6.4: Drug Game: Sensitivity Analysis cont'd

| Game | Case I($*$) | | Case II($**$) | |
|---|---|---|---|---|
| N | Coalition | Payoff | Coalition | Payoff |
| 9 | {F,S} | {500000,500000} | {F,S} | {950000,50000} ($g_1^f = 900000$, $g_2^f = -900000$) |
| 10 | {HI,DHI} | {500000,500000} | {HI,F} | {850000,150000} ($g_1^f = 700000$, $g_2^f = -900000$) |
| 11 | {HI,S} | {500000,500000} | {HI,S} | {950000,50000} ($g_1^f = 900000$, $g_2^f = -900000$) |
| 12 | {HI,S} | {500000,500000} | {HI,S} | {950000,50000} ($g_1^f = 900000$, $g_2^f = -900000$) |

*-$g_i^p = 10^6$, $g_i^f = 0, \forall i$ in all games.

**-$g_i^p = 10^6, \forall i$ in all games.

- **Game 1** (*S-F-HI*)

  For this game, the ***S*** player prefers to join the ***F*** player. The results obtained show that there are no changes in coalition formation if $g_1^p$ of ***HI*** player are reduced. However if the ***HI*** player does not bother about the ***S*** player having at least 500000 more which reflect $g_1^f$, then the coalition of **{*S, HI*}** might form with the allocation of

$\{750000, 250000\}$ .

- **Game 2** (*S-F-DHI*)

  In this game, the ***S*** player prefers to join the ***DHI*** player. However the ***F*** player manages to win in this game if he agrees on the allocation of $\{750000, 250000\}$ upon their coalition.

- **Game 3** (*S-HI-DHI*)

  For this game, the ***HI*** player has the chance to win over the ***DHI*** to form a coalition if he agrees the ***S*** player having 700000 more than him. Hence, the payoff allocation for **{*S, HI*}** is $\{850000, 150000\}$.

- **Game 4** (*DHI-HI-F*)

  For this game, the ***DHI*** player will form a coalition with the ***HI*** player if he can get 400000 more than his opponent. At the same time the ***HI*** player does not bother to have 500000 less than his payoff. Hence the payoff allocation is $\{750000, 250000\}$.

- **Game 5** (*DHI-HI-S*)

  In this game, there is no other way that the ***HI*** player can form a coalition with the ***DHI*** player as the ***DHI*** player is only interested to form a coalition with the ***S*** player. However the present of ***HI*** player in this game may drive to an allocation of $\{950000, 50000\}$ for

$\{\boldsymbol{DHI, S}\}$.

- <u>**Game 6** (*DHI-F-S*</u>)

  It also proven in this game, there is no other way the ***F*** player can form a coalition with the ***DHI*** player in the present of ***S*** player. However the present of ***F*** player in this game may drive to allocation of $\{950000, 50000\}$ for $\{\boldsymbol{DHI, S}\}$.

- <u>**Game 7** (*F-HI-DHI*</u>)

  In this game, the ***HI*** will have to agree with the payoff allocation of $\{950000, 50000\}$ to win the game.

- <u>**Game 8** (*F-HI-S*</u>)

  For this game, there is no other way the ***HI*** player could form a coalition with the ***F*** player. However the present of ***HI*** player in this game may drive to allocation of $\{950000, 50000\}$ for $\{\boldsymbol{F, S}\}$.

- <u>**Game 9** (*F-DHI-S*</u>)

  For this game, there is no other way the ***DHI*** player could form a coalition with the ***F*** player. However the present of ***DHI*** player in this game may drive to allocation of $\{950000, 50000\}$ for $\{\boldsymbol{F, S}\}$.

- <u>**Game 10** (*HI-F-DHI*</u>)

  In this game, the ***F*** player will have to agree with payoff allocation of

$\{850000, 150000\}$ to win the game and form a coalition with the ***HI*** player.

- **Game 11** (*HI-F-S*)

  For this game, there is no other way the ***F*** player can form a coalition with the ***HI*** player. However the present of ***F*** player in this game may drive to allocation of $\{950000, 50000\}$ for **{*HI, S*}**.

- **Game 12** (*HI-S-DHI*)

  For this game, there is no other way the ***DHI*** player can form a coalition with the ***HI*** player. However the present of ***DHI*** player in this game may drive to allocation of $\{950000, 50000\}$ for **{*HI,S*}**.

It can be seen that fairness concerns is weighed more than profit target when in most of the games, the changes in coalition form only happened when the parameter values in $g_1^f$ in the model has changed. However for any coalition that has not changed, the present of the other players in the games has brought variation in the allocation of the profits.

### 6.4.2 Land Game

In the application of the fairness model to the land game, the simulation experiments were conducted in which players differs in their preferences which are distinguished as $(\boldsymbol{S})$, $(\boldsymbol{DHI})$, $(\boldsymbol{F})$ and $(\boldsymbol{HI})$ player as displayed from table 6.5 to 6.7. As the maximum target of every player in the land game is different, the total games to be played is more than the drug game. By considering every possibility of the preferences, there are 24 games to be played to see which coalition will be formed and how payoffs be allocated among players. The land game model is then solved by using Lingo programming (A.2.2).

Table 6.5: Experiments on Asymmetrical Preferences

| **Player** | **Game** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| N | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | S | S | S | S | S | S | DHI | DHI |
| 2 | F | HI | F | DHI | HI | DHI | HI | F |
| 3 | HI | F | DHI | F | DHI | HI | F | HI |

Table 6.6: Experiments on Asymmetrical Preferences cont'd

| **Player** | **Game** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| N | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | DHI | DHI | DHI | DHI | F | F | F | F |
| 2 | HI | S | F | S | HI | DHI | HI | S |
| 3 | S | HI | S | F | DHI | HI | S | HI |

Table 6.7: Experiments on Asymmetrical Preferences cont'd

| **Player** | **Game** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| N | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 1 | F | F | HI | HI | HI | HI | HI | HI |
| 2 | DHI | S | F | DHI | F | S | DHI | S |
| 3 | S | DHI | DHI | F | S | F | S | DHI |

In the land game, the coalition of player 1 and 3 generate more profits than the coalition of player 1 and 2. Player 1 has some authority in this game as player 1 has 10000 on his own and also the power to decide to either join up with player 2 or 3. The validity of the model is tested by considering only $g_1^p$ in this game. It is assumed that every player's desire is to reach maximum target of their $g_1^p$. For this case, both player 1 and 3 will strive for 30000 and player 2 for 20000. It has shown that the results obtained are the same as what suggested by the core solutions $\{X_1, X_2, X_3\} = \{300000, 0, 20000\}$, where the player 2 is left out of the game. Further, $g_1^f$ be considered in the model in order to induce more fairness payoffs allocation between coalition members. For this reason, the series of experiment are conducted to observe the effects of when fairness concerns is imply in human decision making model in this games. The results is expected to suggest other possible coalition formation that may be claim as fair with the suggested allocation of payoff. For $g_1^p$, it is natural to assume that every player will have the maximum target and for $g_1^f$, every player will try to reach the equality target. By keeping the goals constant, the fairness asymmetry in players are represented by the weight values attached to the deviation variables of $g_1^f$ as can be seen in Table 6.1. The result shown in Table 6.8.

Table 6.8: Land Game

| Game | Player | | | Result | |
|---|---|---|---|---|---|
| N | 1 | 2 | 3 | Coalition | Payoff |
| 1 | S | F | HI | {S,HI} | {15000,15000} |
| 2 | S | HI | F | {S,F} | {15000,15000} |
| 3 | S | F | DHI | {S,DHI} | {15000,15000} |
| 4 | S | DHI | F | {S,F} | {15000,15000} |
| 5 | S | HI | DHI | {S,DHI} | {15000,15000} |
| 6 | S | DHI | HI | {S,DHI} | {10000,10000} |
| 7 | DHI | HI | F | {DHI,F} | {15000,15000} |
| 8 | DHI | F | HI | {DHI,HI},{DHI,F} | {15000,15000},{10000,10000}* |
| 9 | DHI | HI | S | {DHI,S} | {15000,15000} |
| 10 | DHI | S | HI | {DHI,S} | {10000,10000} |
| 11 | DHI | F | S | {DHI,S} | {15000,15000} |
| 12 | DHI | S | F | {DHI,F} | {15000,15000} |
| 13 | F | HI | DHI | {F,DHI} | {15000,15000} |
| 14 | F | DHI | HI | {F,DHI} | {10000,10000} |
| 15 | F | HI | S | {F,S} | {15000,15000} |
| 16 | F | S | HI | {F,S} | {10000,10000} |

Table 6.8: Land Game cont'd

| Game | Player | | | Result | |
|---|---|---|---|---|---|
| N | 1 | 2 | 3 | Coalition | Payoff |
| 17 | F | DHI | S | {F,S} | {15000,15000} |
| 18 | F | S | DHI | {F,DHI} | {15000,15000} |
| 19 | HI | F | DHI | {HI,DHI} | {15000,15000} |
| 20 | HI | DHI | F | {HI,F} | {15000,15000} |
| 21 | HI | F | S | {HI,S} | {15000,15000} |
| 22 | HI | S | F | {HI,S} | {10000,10000} |
| 23 | HI | DHI | S | {HI,S} | {15000,15000} |
| 24 | HI | S | DHI | {HI,DHI} | {15000,15000} |

It can be seen from Table 6.8, coalition with a ***HI*** player is never be an option except for game 1. In that case, when a ***S*** player has to choose between a ***HI*** and ***F*** player, he would go for the one who can generate more profit to form a coalition with. Whilst the ***HI*** player prefer to join a coalition with the ***S*** rather than ***F*** player. However, in the present of a ***DHI*** player, the ***HI*** player will only interested to join a coalition with the ***S*** player if it can generate higher profit. Otherwise, the ***DHI*** player would be an option. The ***S*** player would prefer to join a coalition with the ***DHI*** player even though at some coalition, it will generate less profit in comparison to the other coalition. However in the presence of the ***F*** player, it is not. In term of rationality, player 2 is not an option because it generates less profit, therefore the results obtained in game 6 reveals compelling result. However, by taking fairness consideration in the model, player 2 is preferred if he is a ***DHI*** player and the other opponent is a ***HI*** player. Thus, at this point, it might be true to say that the rationality assumption is not really applicable in some games about cooperation. For the ***DHI*** and ***F*** player, they will join a coalition with any players except ***HI*** that can generate more profit.

Similar to drug game, the systematic what-if analysis of the GP model was carried out for each coalition to see whether there is any changes that will change the exist coalition and payoff allocation between players as obtained in table 6.8. The parameter values of $g_1^p$ and $g_1^f$ were investigated to find the satisfice yet feasible break point that will change the exist coalition formation. The summary of this analysis is shown in table 6.9. Case I represent the coalition formation from the various preemptive perceptions of players towards both goals, $g_1^p$ and $g_1^f$. Whilst Case II represent the new coalition formation with the minimum changes in parameter value of $g_1^p$ and $g_1^f$ that will change the coalition formation.

Table 6.9: Land Game: Sensitivity Analysis

| Game | Case I(*) | | Case II(**) | |
|---|---|---|---|---|
| N | Coalition | Payoff | Coalition | Payoff |
| 1 | {S,HI} | {15000,15000} | {S,F} | {13500,6500}<br>($g_2^f = -7000$) |
| 2 | {S,F} | {15000,15000} | {S,F} | {15000,15000} |
| 3 | {S,DHI} | {15000,15000} | {S,DHI} | {15000,15000} |
| 4 | {S,F} | {15000,15000} | {S,F} | {15000,15000} |
| 5 | {S,DHI} | {15000,15000} | {S,DHI} | {15000,15000} |
| 6 | {S,DHI} | {10000,10000} | {S,HI} | {16000,14000}<br>($g_3^f = -2000$) |
| 7 | {DHI,F} | {15000,15000} | {DHI,F} | {15000,15000} |
| 8 | {DHI,HI} | {15000,15000}* | {DHI,F} | {19500,500}<br>($g_1^f = 10000$, $g_2^f = -19000$) |
| 9 | {DHI,S} | {15000,15000} | {DHI,S} | {24500,5500}<br>($g_1^f = 19000$, $g_2^f = -19000$) |
| 10 | {DHI,S} | {10000,10000} | {DHI,HI} | {27500,2500}<br>($g_3^f = -25000$) |
| 11 | {DHI,S} | {15000,15000} | {DHI,S} | {24500,5500} |

Table 6.9: Land Game: Sensitivity Analysis cont'd

| Game | Case I(*) | | Case II(**) | |
|---|---|---|---|---|
| N | Coalition | Payoff | Coalition | Payoff |
| | | | | $(g_1^f = 19000,\ g_2^f = -19000)$ |
| 12 | {DHI,F} | {15000,15000} | {DHI,F} | {15000,15000} |
| 13 | {F,DHI} | {15000,15000} | {F,HI} | {24500,5500} $(g_1^f = 19000,\ g_2^f = -19000)$ |
| 14 | {F,DHI} | {10000,10000} | {F,HI} | {23500,6500} $(g_1^f = 5000,\ g_2^f = -17000)$ |
| 15 | {F,S} | {15000,15000} | {F,S} | {25000,500} $(g_1^f = 20000,\ g_2^f = -19000)$ |
| 16 | {F,S} | {10000,10000} | {F,S} | {19500,500} $(g_1^f = 19000,\ g_3^f = -13000)$ |
| 17 | {F,S} | {15000,15000} | {F,S} | {24500,5500} $(g_1^f = 19000,\ g_2^f = -19000)$ |
| 18 | {F,DHI} | {15000,15000} | {F,DHI} | {24500,5500} $(g_1^f = 19000,\ g_2^f = -19000)$ |
| 19 | {HI,DHI} | {15000,15000} | {HI,DHI} | {24500,5500} $(g_1^f = 19000,\ g_2^f = -19000)$ |

Table 6.9: Land Game: Sensitivity Analysis cont'd

| Game | Case I(*) | | Case II(**) | |
|---|---|---|---|---|
| N | Coalition | Payoff | Coalition | Payoff |
| 20 | {HI,F} | {15000,15000} | {HI,DHI} | {14500,5500} ($g_1^f = 9000,\ g_2^f = -19000$) |
| 21 | {HI,S} | {15000,15000} | {HI,S} | {24500,5500} ($g_1^f = 19000,\ g_2^f = -19000$) |
| 22 | {HI,S} | {10000,10000} | {HI,S} | {19500,500} ($g_1^f = 19000,\ g_3^f = -7000$) |
| 23 | {HI,S} | {15000,15000} | {HI,S} | {24500,5500} ($g_1^f = 19000,\ g_2^f = -19000$) |
| 24 | {HI,DHI} | {15000,15000} | {HI,DHI} | {24500,5500} ($g_1^f = 19000,\ g_2^f = -19000$) |

*-$g_{1,3}^p = 3000, g_2^p = 2000,\ g_i^f = 0, \forall i$ in all games.

**-$g_{1,3}^p = 3000, g_2^p = 2000$

- **Game 1** (*S-F-HI*)

  For this game, the ***S*** player would prefer to join a coalition with the ***HI*** player. However the ***F*** player still can win the game if he agrees with the allocation of $\{13500, 6500\}$. This is unexpected results as it is known the coalition of $\{\boldsymbol{S}, \boldsymbol{F}\}$ generate less profit than $\{\boldsymbol{S}, \boldsymbol{HI}\}$.

- **Game 2** (*S-HI-F*), **Game 3** (*S-F-DHI*), **Game 4** (*S-DHI-F*), **Game 5** (*S-HI-DHI*)

  For all these games, there is no other way that can change the exist coalition formation as well as payoffs allocation.

- **Game 6** (*S-DHI-HI*)

  Although ***HI*** player is not preferred by most of the players, for this coalition this kind of player still manage to win the coalition with ***S*** if agreed with the allocation of $\{16000, 14000\}$.

- **Game 7** (*DHI-HI-F*), **Game 12** (*DHI-S-F*)

  For these games, there is no other way that can change the exist coalition formation as well as payoffs allocation although the maximum changes for both goals, $g_1^p$ and $g_1^f$ have been done.

- **Game 8** (*DHI-F-HI*)

  For this game, the ***HI*** player has the chance to win the game and form a coalition with the ***DHI*** player if he agrees upon the allocation of $\{27500, 2500\}$.

- **Game 9** (*DHI-HI-S*), **Game 11** (*DHI-F-S*)

  In these games, although there is no other way can change the exist coalition formation, however, the changes of other player goals may push to the allocation of $\{24500, 5500\}$ for both coalition.

- **Game 10** (*DHI-S-HI*)

  For this game, the ***DHI*** player would prefer to join a coalition with the ***S*** player. However the ***HI*** player still can win in this game if he agrees upon the allocation of $\{19500, 500\}$.

- **Game 13** (*F-HI-DHI*), **Game 14** (*F-DHI-HI*)

  For both games, the ***F*** player would prefer to join a coalition with the ***DHI*** player. However the ***HI*** player still can win in this game if he agrees with the allocation of $\{24500, 5500\}$ and $\{23500, 6500\}$ respectively.

- **Game 15** (*F-HI-S*), **Game 16** (*F-S-HI*), **Game 17** (*F-DHI-S*)

  For these games, the changes of the other players goals do not affect the coalition formation, however they do affect the payoffs allocation in which the payoff division are change to $\{25000, 5000\}$, $\{19500, 500\}$ and $\{24500, 5500\}$ respectively.

- **Game 18** (*F-S-DHI*)

  In these games, although there is no changes in coalition formation but the changes of other player goals may push to the allocation of

$\{24500, 5500\}$ for both coalition.

- **Game 20** (*HI-DHI-F*)

  For this game, the ***HI*** player would prefer to join a coalition with the ***F*** player. However the ***DHI*** player still can win in this game if he agrees with the allocation of $\{14500, 5500\}$.

- **Game 19** (*HI-F-DHI*), **Game 21** (*HI-F-S*), **Game 23** (*HI-DHI-S*), **Game 24** (*HI-S-DHI*)

  For these games, the changes of the other players goals do not affect the coalition formation, however the payoff allocations do change to $\{24500, 5500\}$ for each coalition.

- **Game 22** (*HI-S-F*)

  There is also no changes in coalition formation in this game but the changes of other player goals may push to the allocation of $\{24500, 5500\}$ for both coalition.

## 6.5 Concluding Remarks

This chapter has introduced one approach to incorporate fairness into normative models of cooperative games that seek to answer the questions which coalition is likely to form and how are the payoffs distributed amongst its members in the sense of fairness. This chapter has investigated the results of asymmetry in fairness perception between players in a game that can be useful for solving cooperative games. More precisely, it has tried to demonstrate how individual preferences in fairness perception can be good to some corporative game situations which is not solely focus on profit maximising. In order to do so, this chapter distiguished two types of goals, which are profit and fairness goal in human decision making in cooperative games. It seems that different individuals have different preferences for fairness versus profit. It is possible to state that cooperation can be represented in terms of tradeoffs in pleasure (profit) and preference(fairness). It cannot be neglected that the more pleasure one player derived from a game, the more cooperative he is likely to be. However, it is believed that there should be some sort of common goals, shared values or even a kind of reciprocation so that the successful coalition might form. The role of trust in cooperation is mainly in the elimination of fear of being betrayed or not being reciprocated. The two main conditions to be met besides trust to facilitate cooperative actions are having a common goal or sharing some values and expecting others to cooperate. In general, one could say that cooperation occurs when there is a non-mutually exclusive goal in which everyone wants to reach a better situation. The presence of mentalising concept in this model is believed to help us to understand the fairness concept in cooperation.

This chapter found at first stage, varying the fairness preferences among players, while holding the monetary goal fixed, influences the coalition formation. Then in the second stage, the minimum value of both goals is searched to find the reference points the coalition formation will change. Through numerical examples of drug and land games, it has shown that some coalitions surprisingly may change their mind about cooperation even though at some points, they are not leading to maximise profit. For instance in land game, the player 2 has joined the coalition even though that coalition only generate less profit in comparison with other coalition. These situations observed in coalitions such as $\{S, HI\}$, $\{DHI, F\}$, $\{F, S\}$, $\{HI, DHI\}$ and $\{HI, S\}$.

By considering mentalising concept in the fairness model of human cooperation, it can thus resolve the contradiction between the social approach and economic models of social cooperation, retaining the analytical of cooperative games and the rational player while incorporating the collective, normative and cultural characteristics stressed in models of norm compliance.

The fairness present of social role in this coalition gives both normative and positive aspects. The positive aspects require every player's payoff is independent of each other, which is rare. Social roles are profoundly interdependent. For instance, a player may have conflict between his own desire and his role in public relations.

# 7

# Discussions and Conclusions

The inconsistency of fairness between conventional economic theory and experimental studies by real people makes this avenue of research intriguing. It is believed that the lack of incorporating reasoning about fairness is part of the cause. The purpose of this research was to provide a fairness modeling perspective in economic games about cooperation based on an understanding of the results presented by behavioral studies, and the theories available from the fields of cognitive neuroscience and psychology.

In a large part of the existing literature on fairness modeling, every human is assumed to adopt the same thinking about fairness in the sense that a the degree of fairness associated to a given solution is assumed to be evaluated in the same manner by every person who is positioned in the shoes of a particular player in the game. A challenge to model an agreed upon formal measure of fairness model in relation to economic decision making has brought researchers to propose several solutions such as inequity aversion model (Fehr and Schmidt, 1999), equity-reciprocity competition model (Bolton and Ockenfels, 2000), distributional and peer-induced fairness model (Ho and Su, 2009) and pleasure model (Haselhuhn and Mellers, 2005). The first two models imply a contributing behaviour which depends on fixed preferences over payoff distributions, regardless of whether the other players have done anything at all. While this research has proposed human fairness model in which asymmetrical fairness is viewed as a tradeoff between players to reach decision. The proposed methodology was able to establish agreement that may be used as an alternative to come up with a fair solution. The simulation results obtained show some similarities to the experimental finding observed in real subjects as in study by (Haselhuhn and Mellers, 2005).

To the best of our knowledge, Goal Programming (GP) has not yet been explored as a way to construct descriptive models of human decision making between players in economic coordination games such as the Ultimatum Game. For constructing the GP framework for such games, we have aimed to capture, at a high-level, some key concepts on human decision making put forward in theoretical cognitive neuroscience including the concepts of goals, efficient biological computation, theory of mind, and reward prediction error mechanisms. A Chebychev GP model was proposed as a way to represent how a decision maker thinks about fairness in these games. The model considers the trade-off between a player's own desires and the desires she thinks the other player has. We have considered the results from the field of (evolutionary) psychology in order to specify the type of desires (or criteria) players consider, which we have named monetary pleasure, fear of rejection, reputation, and emotional criteria, respectively.

The GP modeling approach was chosen to demonstrate that individuals will typically perceive the fairness of a solution as an individual judgement, at any one time ruled by both congnitive and subconscious (emotional) reward-prediction error signals. It stresses the importance of modelling based on distributions about fairness judgements, and of having uncertainty about the other player's judgements. Indeed, if the ToM model in the UG is modeled 'correctly', i.e. based on the real goals values and weight of the other player, then all offers made would be accepted. This does not occur in practise. Hence, it is the information assymmetry and uncertainty that results in offers being rejected, which reduces the social welfare. In general, not the differences in fairness perception, but rather the fact that some knowledge remains private to one of the players, causes failures in cooperation. Neuroscience further suggests that some of this knowledge remains even hidden from the conscious mind of the player.

We have also considered a GP modeling approach to analyse cooperative games in which a player needs to select between two potential partners to form a coalition. Which coalition is likely to form and how the payoffs are being distributed amongst its members is affected by fairness considerations. The research has shown that assuming an asymmetry in fairness judgements produces different solutions to the classic concepts of cooperative game theory in which such asymmetries are not captured. The fairness of cooperation in the Drug Game and Land Game is represented as a tradeoff between the goals of pleasure (profit) and preference (fairness). The minimum value of both goals is then searched to find the stable payoff for coalition members. It can be deduced from this model which types of players are more likely to cooperate and what the allocation of relative profits would be. Asymmetry in fairness perception can lead to stable cooperation with respect to individual rationality. For example, a 90/10 division of total payoffs can be considered as a fair allocation for certain types of players. Also, it shows that in reality it sometimes pays to be to be not the most powerful player.

The field of cognitive neuroscience should be considered as a legitimate and important area for future research, which will allow us to more fully understand human behavior in important contexts, and use this knowledge to adapt decision models. This dissertation asserts that the ideas in cognitive neuroscience as described by (Montague, 2007) find good resonance with the behavioral theory of Herbert Simon who introduced the concept of satisficing as an alternative model of rational choice, i.e. a model of bounded rationality. The link to Goal Programming is hence derived.

It must be stressed that the theory of mind itself is constantly evolving, as well as insights into exactly what activity and processes in various areas of the brain actually mean. It should not be forgotten that this concept is essentially subjective and cannot directly 'prove' a posited relationship between game strategies. Nevertheless, better and more objective measurement and observation, as can be provided by neuroscience in many cases, allows us to get closer to understanding what really happens in response to economic games. This line of this research requires more advanced and extensive description of the human way of thinking, which subsequently may be integrated in multi-objective problems.

The field of psychology is also important in understanding human decision making processes and is shown to be applicable in the context of Goal Programming. While (Fehr and Schmidt, 1999)'s model caters for fairness between all possible pairs of players, this research model fairness that allows asymmetrical concerns between multiple players and it can be played simultaneously rather than sequentialy. Our framework, though in different way, also incorporate (Rabin, 1993) inference theory that one's tendency to cooperate depends on beliefs about the other player's behaviour. Based on the valuable information derived from the real experiments in the fields evolutionary psychology and neuro-economics, it turns out that righteous anger and fear of rejection, concerns about reputation and also sympathy or trust may thus influence decisions. By including these features in the GP modeling and also distinguishing between the players' model of themselves and a model they construct of the other players, the human decision making model is developed.

It may be hoped that this dissertation offers a step towards a broader, more valuable interpretation of fairness. The success of the rudimentary fitting of the Drug and Land game suggests that more formal quantitative models incorporating the asymmetry of fairness judgements by indivuals when acting the role of a particular player, are worth exploring. The model of the Ultimatum Game stresses the importance of hidden knowledge about fairness as a source of uncertainty that impacts game results.

The study has numerous limitations. For example, in UG experiment, keeping goals constant but assuming a range of values for weights, a series of computer experiments produced distributions of accepted and rejected offers that show some similarity with the distributions observed from real experiments with human subjects. Such simulations can be improved from further research. Reverse engineering the model, for example, by asking for a better fit with real data, could lead to information about ranges or distributions of likely goal values and weights that people use, assuming that the logic of the GP model has some degree of accuracy. One could also asks people how they would set the different goals presented, and choose their weights. One particular area of interest is to explore games that are repeated. Indeed, fairness concerns could be interpreted as emotional responses that make sure players do well in the long term in games on cooperation that can repeat and where players would switch roles.

It is not to claim that the GP approach presented has value in terms of accurately modeling real decision processes in the brain. It just offers a simple approach to model the decision processes in an abstract manner that does seem to incorporate to some degree some important concepts from cognitive neuroscience about how we make decisions. While GP on itself does offer more flexibility and power towards modeling decision problems, we have intentionally used a very simple form that can serve as a starting point for further refinements.

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# A
# Appendix

## Appendix A: Chapter 5

### A.1.1 Ultimatum Game of Proposer Model-Lingo Programming

```
MODEL: !UG GAME; !PROPOSER;

SETS: GAME/R1..R576/:MYPAYOFF, YOURPAYOFF,LAMBDA,FN,FP,FNW,FPW;
!PROPOSER, RESPONDER; GOAL/G1,G2,G3,G4/:; !MONETARY PLEASURE, FEAR
OF REJECTION, REPUTATION, SYMPATHY; GAME_GOAL(GAME, GOAL):
MY_TARGET, YOUR_TARGET, MN, MP, YN, YP, MW, YW; ENDSETS

DATA: !IMPORT DATA FROM EXCEL;
MY_TARGET, YOUR_TARGET, MW,
YW=@OLE('N:\Configs\Desktop\myphdfinal\chapter
6\UG-PROPOSERTORESPONDER.XLSX',
'MY_TARGET','YOUR_TARGET','MW','YW');

!EXPORT DATA TO EXCEL; @OLE('N:\Configs\Desktop\myphdfinal\chapter
6\UG-PROPOSERTORESPONDER.XLSX', 'YOURPAYOFF','LAMBDA')=
YOURPAYOFF,LAMBDA;

ENDDATA

!OBJECTIVE FUNCTION; MIN=@SUM(GAME(I):LAMBDA(I));

!PROPOSER PROFIT GOALS; @FOR(GAME(I):
    @FOR(GAME_GOAL(I,J): MYPAYOFF(I) + MN(I,J) - MP(I,J) = MY_TARGET(I,J)));

!TOM RESPONDER GOALS; @FOR(GAME(I):
    @FOR(GAME_GOAL(I,J): YOURPAYOFF(I) + YN(I,J) - YP(I,J) = YOUR_TARGET(I,J)));

!MINIMISING PROPOSER PROFIT DEVIATIONS;

@FOR(GAME(I):MW(I,1)*MN(I,1)+MW(I,2)*MP(I,2)+MW(I,3)*(MN(I,3)+MP(I,3))
+MW(I,4)*MP(I,4) <=LAMBDA(I));

!MINIMISING TOM RESPONDER DEVIATIONS;

@FOR(GAME(I):YW(I,1)*YP(I,1)+YW(I,2)*YP(I,2)+YW(I,3)*YN(I,3)+YW(I,4)*YN(I,4)
<=LAMBDA(I));

!HARD CONSTRAINT;

@FOR(GAME(I): MYPAYOFF(I)+YOURPAYOFF(I)=10);
END
```

### A.1.2 Ultimatum Game of Responder Model-Lingo Programming

```
MODEL: !ULTIMATUM GAME; !RESPONDER;

SETS: GAME/R1..R576/:MYPAYOFF, YOURPAYOFF,LAMBDA; !RESPONDER,
PROPOSER; GOAL/G1,G2,G3,G4/: ; !MONETARY PLEASURE, FEAR OF
REJECTION, REPUTATION, SYMPATHY; GAME_GOAL(GAME, GOAL): MY_TARGET,
YOUR_TARGET, MN, MP, YN, YP, MW, YW; ENDSETS

DATA: !IMPORT DATA FROM EXCEL; MY_TARGET, YOUR_TARGET, MW, YW
=@OLE('N:\Configs\Desktop\myphdfinal\chapter
6\UG-PROPOSERTORESPONDER.XLSX',
'MY_TARGET','YOUR_TARGET','MW','YW');
```

```
!EXPORT DATA TO EXCEL; @OLE('N:\Configs\Desktop\myphdfinal\chapter
6\UG-PROPOSERTORESPONDER.XLSX',  'MYPAYOFF','LAMBDA')= MYPAYOFF,
LAMBDA; ENDDATA

!OBJECTIVE FUNCTION; MIN=@SUM(GAME(I):LAMBDA(I));

!RESPONDER GOALS; @FOR(GAME(I):
    @FOR(GAME_GOAL(I,J): MYPAYOFF(I) + MN(I,J) - MP(I,J) = MY_TARGET(I,J)));

!TOM PROPOSER GOALS; @FOR(GAME(I):
    @FOR(GAME_GOAL(I,J): YOURPAYOFF(I) + YN(I,J) - YP(I,J) = YOUR_TARGET(I,J)));

!MINIMISING RESPONDER DEVIATIONS;

@FOR(GAME(I):MW(I,1)*MN(I,1)+MW(I,2)*MP(I,2)+MW(I,3)*(MN(I,3)+MP(I,3))+MW(I,4)*MP(I,4)
<=LAMBDA(I));

!MINIMISING TOM PROPOSER DEVIATIONS;

@FOR(GAME(I):YW(I,1)*YP(I,1)+YW(I,2)*YP(I,2)+YW(I,3)*YN(I,3)+YW(I,4)*YN(I,4)
<=LAMBDA(I));

!HARD CONSTRAINT;

@FOR(GAME(I): MYPAYOFF(I)+YOURPAYOFF(I)=10);

END
```

### A.1.3 Ultimatum Game between Proposer and Responder: VB Code

```
Sub UG6()
'INPUT VARIABLES
Dim RAccept As Double
Dim POffer As Double
Dim MAXGP As Variant
Dim MAXGR As Variant
Dim MINGP As Variant
Dim MINGR As Variant

Dim PCounter As Long
Dim RCounter As Long


'OUTPUT VARIABLES
Dim AcceptOffer As Integer '0 if not accepted, 1 if accepted
Dim GProposerAccept As Integer


'COLLECT OUTPUT STATS
Dim OfferAcceptedCountArray(10) As Long
Dim OfferRejectedCountArray(10) As Long
Dim ProposerHighestG1AcceptedCountArray(10) As Long
Dim ProposerHighestG2AcceptedCountArray(10) As Long
Dim ProposerHighestG3AcceptedCountArray(10) As Long
Dim ProposerHighestG4AcceptedCountArray(10) As Long

Dim ProposerHighestG1RejectedCountArray(10) As Long
Dim ProposerHighestG2RejectedCountArray(10) As Long
Dim ProposerHighestG3RejectedCountArray(10) As Long
Dim ProposerHighestG4RejectedCountArray(10) As Long

Dim ResponderHighestG1AcceptedCountArray(10) As Long
Dim ResponderHighestG2AcceptedCountArray(10) As Long
Dim ResponderHighestG3AcceptedCountArray(10) As Long
Dim ResponderHighestG4AcceptedCountArray(10) As Long

Dim ResponderHighestG1RejectedCountArray(10) As Long
Dim ResponderHighestG2RejectedCountArray(10) As Long
Dim ResponderHighestG3RejectedCountArray(10) As Long
Dim ResponderHighestG4RejectedCountArray(10) As Long

Dim ProposerLowestG1AcceptedCountArray(10) As Long
Dim ProposerLowestG2AcceptedCountArray(10) As Long
Dim ProposerLowestG3AcceptedCountArray(10) As Long
Dim ProposerLowestG4AcceptedCountArray(10) As Long
```

```
Dim ProposerLowestG1RejectedCountArray(10) As Long
Dim ProposerLowestG2RejectedCountArray(10) As Long
Dim ProposerLowestG3RejectedCountArray(10) As Long
Dim ProposerLowestG4RejectedCountArray(10) As Long

Dim ResponderLowestG1AcceptedCountArray(10) As Long
Dim ResponderLowestG2AcceptedCountArray(10) As Long
Dim ResponderLowestG3AcceptedCountArray(10) As Long
Dim ResponderLowestG4AcceptedCountArray(10) As Long

Dim ResponderLowestG1RejectedCountArray(10) As Long
Dim ResponderLowestG2RejectedCountArray(10) As Long
Dim ResponderLowestG3RejectedCountArray(10) As Long
Dim ResponderLowestG4RejectedCountArray(10) As Long


Dim Bin As Integer


   For PCounter = 1 To 576 Step 1
      'POffer = Worksheets("Sheet2").Cells(PCounter + 1, 2)
      POffer = Worksheets("Sheet1").Cells(PCounter + 1, 44)
      MAXGP = Worksheets("Sheet1").Cells(PCounter + 1, 46)
      MINGP = Worksheets("Sheet1").Cells(PCounter + 1, 45)

    For RCounter = 1 To 576 Step 1
   'RAccept = Worksheets("Sheet2").Cells(RCounter + 1, 1)
      RAccept = Worksheets("Sheet1").Cells(RCounter + 1, 21)
      MAXGR = Worksheets("Sheet1").Cells(RCounter + 1, 23)
      MINGR = Worksheets("Sheet1").Cells(PCounter + 1, 22)

         If POffer >= RAccept Then
             AcceptOffer = 1
             Bin = POffer
             OfferAcceptedCountArray(Bin) = OfferAcceptedCountArray(Bin) + 1

                If MAXGP = "G1" Then
                        ProposerHighestG1AcceptedCountArray(Bin) = ProposerHighestG1AcceptedCountArray(Bin) + 1

                 Else

                        If MAXGP = "G2" Then
                              ProposerHighestG2AcceptedCountArray(Bin) = ProposerHighestG2AcceptedCountArray(Bin) + 1

                        Else

                              If MAXGP = "G3" Then
                                   ProposerHighestG3AcceptedCountArray(Bin) = ProposerHighestG3AcceptedCountArray(Bin) + 1

                              Else

                                    ProposerHighestG4AcceptedCountArray(Bin) = ProposerHighestG4AcceptedCountArray(Bin) + 1
                              End If
                        End If
                  End If

                  If MINGP = "G1" Then
                        ProposerLowestG1AcceptedCountArray(Bin) = ProposerLowestG1AcceptedCountArray(Bin) + 1

                 Else

                        If MINGP = "G2" Then
                              ProposerLowestG2AcceptedCountArray(Bin) = ProposerLowestG2AcceptedCountArray(Bin) + 1

                        Else

                              If MINGP = "G3" Then
                                   ProposerLowestG3AcceptedCountArray(Bin) = ProposerLowestG3AcceptedCountArray(Bin) + 1

                              Else

                                    ProposerLowestG4AcceptedCountArray(Bin) = ProposerLowestG4AcceptedCountArray(Bin) + 1
                              End If
                        End If
                  End If


                  If MAXGR = "G1" Then
                  ResponderHighestG1AcceptedCountArray(Bin) = ResponderHighestG1AcceptedCountArray(Bin) + 1

                  Else

                       If MAXGR = "G2" Then
```

```
                ResponderHighestG2AcceptedCountArray(Bin) = ResponderHighestG2AcceptedCountArray(Bin) + 1

              Else

                   If MAXGR = "G3" Then
                      ResponderHighestG3AcceptedCountArray(Bin) = ResponderHighestG3AcceptedCountArray(Bin) + 1

                    Else

                    ResponderHighestG4AcceptedCountArray(Bin) = ResponderHighestG4AcceptedCountArray(Bin) + 1
                    End If
              End If
        End If

        If MINGR = "G1" Then
        ResponderLowestG1AcceptedCountArray(Bin) = ResponderLowestG1AcceptedCountArray(Bin) + 1

        Else

             If MINGR = "G2" Then
                ResponderLowestG2AcceptedCountArray(Bin) = ResponderLowestG2AcceptedCountArray(Bin) + 1

              Else

                   If MINGR = "G3" Then
                      ResponderLowestG3AcceptedCountArray(Bin) = ResponderLowestG3AcceptedCountArray(Bin) + 1

                    Else

                    ResponderLowestG4AcceptedCountArray(Bin) = ResponderLowestG4AcceptedCountArray(Bin) + 1
                    End If
              End If
        End If


Else
     AcceptOffer = 0
     Bin = POffer
     OfferRejectedCountArray(Bin) = OfferRejectedCountArray(Bin) + 1

     If MAXGP = "G1" Then
        ProposerHighestG1RejectedCountArray(Bin) = ProposerHighestG1RejectedCountArray(Bin) + 1

      Else
           If MAXGP = "G2" Then
              ProposerHighestG2RejectedCountArray(Bin) = ProposerHighestG2RejectedCountArray(Bin) + 1

            Else

                If MAXGP = "G3" Then
                   ProposerHighestG3RejectedCountArray(Bin) = ProposerHighestG3RejectedCountArray(Bin) + 1

                Else

                    ProposerHighestG4RejectedCountArray(Bin) = ProposerHighestG4RejectedCountArray(Bin) + 1
                End If
            End If
       End If


       If MINGP = "G1" Then
        ProposerLowestG1RejectedCountArray(Bin) = ProposerLowestG1RejectedCountArray(Bin) + 1

      Else
           If MINGP = "G2" Then
              ProposerLowestG2RejectedCountArray(Bin) = ProposerLowestG2RejectedCountArray(Bin) + 1

            Else

                If MINGP = "G3" Then
                   ProposerLowestG3RejectedCountArray(Bin) = ProposerLowestG3RejectedCountArray(Bin) + 1

                Else

                    ProposerLowestG4RejectedCountArray(Bin) = ProposerLowestG4RejectedCountArray(Bin) + 1
                End If
            End If
       End If

    If MAXGR = "G1" Then
       ResponderHighestG1RejectedCountArray(Bin) = ResponderHighestG1RejectedCountArray(Bin) + 1

    Else
```

```
                If MAXGR = "G2" Then
                   ResponderHighestG2RejectedCountArray(Bin) = ResponderHighestG2RejectedCountArray(Bin) + 1

                Else

                   If MAXGR = "G3" Then
                      ResponderHighestG3RejectedCountArray(Bin) = ResponderHighestG3RejectedCountArray(Bin) + 1

                    Else

                      ResponderHighestG4RejectedCountArray(Bin) = ResponderHighestG4RejectedCountArray(Bin) + 1
                    End If
                 End If
          End If

          If MINGR = "G1" Then
             ResponderLowestG1RejectedCountArray(Bin) = ResponderLowestG1RejectedCountArray(Bin) + 1

          Else
                If MINGR = "G2" Then
                   ResponderLowestG2RejectedCountArray(Bin) = ResponderLowestG2RejectedCountArray(Bin) + 1

                Else

                   If MINGR = "G3" Then
                      ResponderLowestG3RejectedCountArray(Bin) = ResponderLowestG3RejectedCountArray(Bin) + 1

                    Else

                      ResponderLowestG4RejectedCountArray(Bin) = ResponderLowestG4RejectedCountArray(Bin) + 1
                    End If
                 End If
          End If


          End If
       Next RCounter
Next PCounter


For Bin = 1 To 10 Step 1
'Worksheets("Sheet3").Cells(4, Bin) = OfferAcceptedCountArray(Bin)
'Worksheets("Sheet3").Cells(5, Bin) = OfferRejectedCountArray(Bin)
Worksheets("Sheet3").Cells(22, Bin) = OfferAcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(23, Bin) = OfferRejectedCountArray(Bin)
Worksheets("Sheet3").Cells(25, Bin) = ProposerHighestG1AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(29, Bin) = ProposerHighestG2AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(33, Bin) = ProposerHighestG3AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(37, Bin) = ProposerHighestG4AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(26, Bin) = ProposerHighestG1RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(30, Bin) = ProposerHighestG2RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(34, Bin) = ProposerHighestG3RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(38, Bin) = ProposerHighestG4RejectedCountArray(Bin)

Worksheets("Sheet3").Cells(27, Bin) = ResponderHighestG1AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(31, Bin) = ResponderHighestG2AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(35, Bin) = ResponderHighestG3AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(39, Bin) = ResponderHighestG4AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(28, Bin) = ResponderHighestG1RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(32, Bin) = ResponderHighestG2RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(36, Bin) = ResponderHighestG3RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(40, Bin) = ResponderHighestG4RejectedCountArray(Bin)

Worksheets("Sheet3").Cells(42, Bin) = ProposerLowestG1AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(46, Bin) = ProposerLowestG2AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(50, Bin) = ProposerLowestG3AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(54, Bin) = ProposerLowestG4AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(43, Bin) = ProposerLowestG1RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(47, Bin) = ProposerLowestG2RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(51, Bin) = ProposerLowestG3RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(55, Bin) = ProposerLowestG4RejectedCountArray(Bin)

Worksheets("Sheet3").Cells(44, Bin) = ResponderLowestG1AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(48, Bin) = ResponderLowestG2AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(52, Bin) = ResponderLowestG3AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(56, Bin) = ResponderLowestG4AcceptedCountArray(Bin)
Worksheets("Sheet3").Cells(45, Bin) = ResponderLowestG1RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(49, Bin) = ResponderLowestG2RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(53, Bin) = ResponderLowestG3RejectedCountArray(Bin)
Worksheets("Sheet3").Cells(57, Bin) = ResponderLowestG4RejectedCountArray(Bin)

Next Bin
```

```
'For Bin = 1 To 4 Step 1
'Worksheets("Sheet3").Cells(24, Bin) = ProposerHighestG1AcceptedCountArray(Bin)
'Worksheets("Sheet3").Cells(25, Bin1) = GPRejectedCountArrayMax(Bin1)
'Next Bin

End Sub
```

### A.1.4 Dictator Game of Proposer Model-Lingo Programming

```
MODEL: !DG GAME; !PROPOSER;

SETS: GAME/R1..R42/:MYPAYOFF, YOURPAYOFF,LAMBDA,FN,FP,FNW,FPW;
!PROPOSER, RESPONDER; GOAL/G1,G2,G3,G4/:; !MONETARY PLEASURE, FEAR
OF REJECTION, REPUTATION, SYMPATHY; GAME_GOAL(GAME, GOAL):
MY_TARGET, YOUR_TARGET, MN, MP, YN, YP, MW, YW; ENDSETS

DATA: !IMPORT DATA FROM EXCEL; MY_TARGET, YOUR_TARGET, MW,
YW=@OLE('N:\Configs\Desktop\myphdfinal\chapter
6\UG-PROPOSERTORESPONDER.XLSX',
'MY_TARGET','YOUR_TARGET','MW','YW');

!EXPORT DATA TO EXCEL; @OLE('N:\Configs\Desktop\myphdfinal\chapter
6\DG-PROPOSERTORESPONDER.XLSX', 'YOURPAYOFF','LAMBDA')=
YOURPAYOFF,LAMBDA;

ENDDATA

!OBJECTIVE FUNCTION; MIN=@SUM(GAME(I):LAMBDA(I));

!PROPOSER PROFIT GOALS; @FOR(GAME(I):
    @FOR(GAME_GOAL(I,J): MYPAYOFF(I) + MN(I,J) - MP(I,J) = MY_TARGET(I,J)));

!TOM RESPONDER GOALS; @FOR(GAME(I):
    @FOR(GAME_GOAL(I,J): YOURPAYOFF(I) + YN(I,J) - YP(I,J) = YOUR_TARGET(I,J)));

!MINIMISING PROPOSER PROFIT DEVIATIONS;

@FOR(GAME(I):MW(I,1)*MN(I,1)+MW(I,2)*MP(I,2)+MW(I,3)*(MN(I,3)+MP(I,3))
+MW(I,4)*MP(I,4) <=LAMBDA(I));

!MINIMISING TOM RESPONDER DEVIATIONS;

@FOR(GAME(I):YW(I,1)*YP(I,1)+YW(I,2)*YP(I,2)+YW(I,3)*YN(I,3)+YW(I,4)*YN(I,4)
<=LAMBDA(I));

!HARD CONSTRAINT;

@FOR(GAME(I): MYPAYOFF(I)+YOURPAYOFF(I)=10); END
```

### A.1.5 Statistical Analyses

The Statistical analysis is used to support the findings from the GP modelling. It enables the experimenter to come to a decision in an objective way when faced with experimental uncertainty. A test of a particular hypothesis is performed as follows. In any hypothesis test, the conditional probabilities are calculated based on the assumption that the null hypothesis, $H_o$ is true. After carrying out a significance test, some evidence is needed to decide whether or not to reject $H_o$. $H_o$ is rejected if the observed value of the test statistic is larger (or smaller) than a particular critical value. This critical value should be chosen before the observations are taken. However it is possible to make a mistake in two different ways. Firstly, it is possible to get a significant result when the $H_o$ is true. This is called an error of type I. Secondly, it is possible to get a non-significant result when the null hypothesis is false. This is called an error of type II. These can be illustrated in table A.1.5.

Table A.1: Hypothesis Testing

| Decision | $H_o$ is true | $H_o$ is false |
|---|---|---|
| Accept $H_o$ | Correct decision | Type II error |
| Reject $H_o$ | Type I error | Correct decision |

Hypothesis testing consists of five steps:

- Stating the null and the alternative hypothesis. The null hypothesis is the hypothesis to be tested.
- Specifying the significance level, $\alpha$.
- Selecting an appropriate statistical test to compute.
- Identifying the probability distribution of the test statistic and determine the critical region.

- Deciding whether or not to reject the null hypothesis.

In this research, statistical tests of Mann Whitney, $z$-test for Two Proportions and Independence Test have been used to support the findings. All these tests can be reviewed in Chase and Bown (2000) and Hutcheson and Moutinho (2008).

#### A.1.5.1 Mann Whitney Test

Non-parametric statistical technique, the Mann Whitney test are employed for variables measured on an ordinal scale. No assumptions of normality are required for such methods and they are often referred to as distribution-free techniques. Beforehand, the Kolmogorov-Smirnov test is conducted in order to check whether observed values can reasonably be thought to have come from a normally distributed population. In this test, if the results is significant and $H_o$ is being rejected, the observed value is non-normal. In Mann Whitney, the null hypothesis tested is that there is no difference between the two groups, $x$ and $y$ focusing on the median as a measure of central tendency. Therefore rejection could be because either their means, variances or the shape of distributions differ, or any combination of these.

- Stating the hypothesis

  (Two-sided test)

$$H_o : median_x = median_y$$

$$H_a : median_x \neq median_y$$

- All the data values in the two samples taken together should be ranked. Let is, $S_x$ the some of the ranks of the values $x$ and $S_y$ the sum of the ranks of the values $y$. Statistical test, $U$ is computed as:

$$U_x = S_x - \frac{n_x(n_x + 1)}{2}$$

$$U_y = S_y - \frac{n_y(n_y + 1)}{2}$$

$$U = min\{U_x, U_y\}$$

- Identify the critical value, $U_{n_x,n_y,\alpha/2}$

- Reject hypothesis null if $U > U_{n_x,n_y,\alpha/2}$ and do not reject otherwise. In other way, the *p-value* obtained from this test also can be used to decide whether or not to reject $H_o$. If *p-value*$< \alpha_{0.01/2}$, then reject $H_o$.

#### A.1.5.2 Hypothesis Testing for Two Proportion

As the name suggests it is used when comparing the percentages of two groups, $p_1$ and $p_2$.

- State the hypothesis

  (One-sided test)

$$H_o : p_1 = p_2$$

$$H_1 : p_1 < p_2$$

or

$$H_1 : p_1 > p_2$$

- Significance level of 1% is chosen
- Hypothesis tests based on a test statistic called z-score, $z$.
- Analyze Sample Data The calculated value takes a form:

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{p_{pooled}(1 - p_{pooled})[\frac{1}{n_1} + \frac{1}{n_2}]}}$$

$$p_{pooled} = (n_1 p_1 + n_2 p_2)/(n_1 + n_2)$$

- Identify the critical value, $z_{0.01}$
- Reject hypothesis null if $z > z_{0.01}$ and do not reject otherwise. In other way, the *p-value* obtained from this test also can be used to decide whether or not to reject $H_o$. If *p-value*$< \alpha_{0.01}$, then reject $H_o$.

#### A.1.5.3 Independence Test

Hypothesis tests may be performed on contingency tables in order to decide whether or not the effects are present. Effects in a contingency table are defined as relationships between the row and column variables; that is, are the levels of the row variable differentially distributed over levels of the column variables.

- $H_o$: The row and column variables are independent (not associated to each other).

  $H_o$: The row and column variables are related.

- Significance level of 1% is chosen

- Hypothesis tests on contingency tables are based on a statistic called Chi-square, $\chi^2$.

Table A.2: Contingency Table

| | | | | | |
|---|---|---|---|---|---|
| A | B | C | D | E | $Tr_1$ |
| F | G | H | I | J | $Tr_2$ |
| K | L | M | N | O | $Tr_3$ |
| Q | R | S | T | U | $Tr_4$ |
| $Tc_1$ | $Tc_2$ | $Tc_3$ | $Tc_4$ | $Tc_5$ | $T_G$ |

Each cell in table A.2 displays the observed frequencies, $O$ while expected frequency, $E$ can be computed as below:

$$E = \frac{(Tr).(Tc)}{T_G}$$

Next, the Chi-square statistic can be computed as follows:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

- Identify the critical value, $\chi_{0.01/2}, (r-1)(c-1)$
- Reject hypothesis null if $\chi^2 > \chi_{0.01}, (r-1)(c-1)$ and do not reject otherwise. In other way, the *p-value* obtained from this test also can be used to decide whether or not to reject $H_o$. If *p-value*$< \alpha_{0.01/2}$, then reject $H_o$.

All those statistical tests also can be analysed using Statistical Package for the Social Sciences (SPSS) software.

# Appendix B: Chapter 6

## A.2.1 Drug Game Pooling Formulation

```
MODEL:
SETS: GOAL/G1, G2/:; CHAR_FUNC/1, 2, 3, 4, 5, 6, 7/:V; !(1) (2)
(3) (12) (13) (23) (123); POOL/S1, S2, S3, S4, S5, S6, S7/: S,N,P;
!(1) (2) (3) (1,2) (1,3) (2,3) (1,2,3); PLAYER/P1, P2, P3/:W, X,
NP, PP; PLAYER_GOAL(PLAYER, GOAL): TARGET, N_W, P_W;
POOL_PAYOFF(PLAYER, POOL): Y, NF, PF;
ENDSETS

DATA:
!PLAYER_GOAL;
W = 1 1 1; TARGET = 800000 200000
                    600000 100000
                    300000 -100000;  !PAYOFFTARGET, !FAIRNESSTARGET;
N_W = 0.7 0.2
      0.1 0.6
```

```
      0.5 0.35; !NEG WEIGHT PAYOFF, WEIGHT ALPHA;
P_W = 0  0.1
      0  0.3
      0  0.15; !POS WEIGHT PAYOFF, WEIGHT BETA;
V=0,0,0,1000000,1000000,0,1000000;
ENDDATA

!OBJECTIVE FUNCTION; !MIN=EPS; !MAX=@SUM(PLAYER(I):X(I));
!OBJECTIVE FUNCTION;
MIN=@SUM(PLAYER(I):W(I)*(N_W(I,1)*NP(I)))+(@SUM(POOL_PAYOFF(I,J):N_W(I,
2) * NF(I, J) + P_W(I, 2) * PF(I,J)));

!CORE CONDITION;
X(1)>=V(1); X(2)>=V(2); X(3)>=V(3);
X(1)+X(2)+N(4)-P(4)=V(4); X(1)+X(3)+N(5)-P(5)=V(5);
X(2)+X(3)+N(6)-P(6)=V(6); X(1)+X(2)+X(3)+N(7)-P(7)=V(7);

N(1)<=EPS; N(2)<=EPS; N(3)<=EPS; N(4)<=EPS;
N(5)<=EPS; N(6)<=EPS;
N(7)<=EPS; !EPS<=0;

X(1)<=Y(1,1)+Y(1,4)+Y(1,5)+Y(1,7);
X(2)<=Y(2,2)+Y(2,4)+Y(2,6)+Y(2,7);
X(3)<=Y(3,3)+Y(3,5)+Y(3,6)+Y(3,7);

!POOL CONSTRAINTS;
Y(1,1)<=V(1)*S(1);
Y(2,2)<=V(2)*S(2);
Y(3,3)<=V(3)*S(3);
Y(1,4)+Y(2,4)<=V(4)*S(4);
Y(1,5)+Y(3,5)<=V(5)*S(5);
Y(2,6)+Y(3,6)<=V(6)*S(6);
Y(1,7)+Y(2,7)+Y(3,7)<=V(7)*S(7);

!POOL SELECTION;
S(1)+S(4)+S(5)+S(7)=1; !PLAYER 1;
S(2)+S(4)+S(6)+S(7)=1; !PLAYER 2;
S(3)+S(5)+S(6)+S(7)=1; !PLAYER
3; !S(5)=0;

!PROFITGOAL;
```

```
X(1)+NP(1)-PP(1)=TARGET(1,1);
X(2)+NP(2)-PP(2)=TARGET(2,1);
X(3)+NP(3)-PP(3)=TARGET(3,1);

!FAIRNESSGOAL;
!PLAYER 1;
Y(1,4)-Y(2,4)+NF(1,4)-PF(1,4)=TARGET(1,2)*S(4);
Y(1,5)-Y(3,5)+NF(1,5)-PF(1,5)=TARGET(1,2)*S(5);
Y(1,7)-(Y(2,7)+Y(3,7))+NF(1,7)-PF(1,7)=TARGET(1,2)*S(7);

!PLAYER 2;
Y(2,4)-Y(1,4)+NF(2,4)-PF(2,4)=TARGET(2,2)*S(4);
Y(2,6)-Y(3,6)+NF(2,6)-PF(2,6)=TARGET(2,2)*S(6);
Y(2,7)-(Y(1,7)+Y(3,7))+NF(2,7)-PF(2,7)=TARGET(2,2)*S(7);

!PLAYER3;
Y(3,5)-Y(1,5)+NF(3,5)-PF(3,5)=TARGET(3,2)*S(5);
Y(3,6)-Y(2,6)+NF(3,6)-PF(3,6)=TARGET(3,2)*S(6);
Y(3,7)-(Y(1,7)+Y(2,7))+NF(3,7)-PF(3,7)=TARGET(3,2)*S(7);

!POOL SELECTION VARIABLES ARE BINARY; @FOR(POOL:@BIN(S));
END
```

### A.2.2 Land Game Pooling Formulation

```
MODEL:
SETS: GOAL/G1, G2/:; CHAR_FUNC/1, 2, 3, 4, 5, 6, 7/:V; !(1) (2)
(3) (12) (13) (23) (123); POOL/S1, S2, S3, S4, S5, S6, S7/: S,N,P;
!(1) (2) (3) (1,2) (1,3) (2,3) (1,2,3); PLAYER/P1, P2, P3/:W, X,
NP, PP; PLAYER_GOAL(PLAYER, GOAL): TARGET, N_W, P_W;
POOL_PAYOFF(PLAYER, POOL): Y, NF, PF;
ENDSETS

DATA: !PLAYER_GOAL;
W = 1 1 1; TARGET = 20000 10000
                    10000 -10000
                    15000 0;  !PAYOFFTARGET, !FAIRNESSTARGET;
N_W = 0.7 0.2
      0.1 0.6
      0.5 0.35; !NEG WEIGHT PAYOFF, WEIGHT ALPHA;
P_W = 0  0.1
```

```
      0  0.3
      0  0.15; !POS WEIGHT PAYOFF, WEIGHT BETA;
V=10000,0,0,20000,30000, 0,30000;
ENDDATA

!OBJECTIVE FUNCTION; !MIN=EPS; !MAX=@SUM(PLAYER(I):X(I));
!OBJECTIVE FUNCTION; MIN=@SUM(PLAYER(I):W(I)*(N_W(I,1)*NP(I)))+
(@SUM(POOL_PAYOFF(I,J):N_W(I, 2) * NF(I, J) + P_W(I, 2) * PF(I,
J)));

!CORE CONDITION;
X(1)>=V(1); X(2)>=V(2); X(3)>=V(3);
X(1)+X(2)+N(4)-P(4)=V(4); X(1)+X(3)+N(5)-P(5)=V(5);
X(2)+X(3)+N(6)-P(6)=V(6); X(1)+X(2)+X(3)+N(7)-P(7)=V(7);

N(4)<=EPS; N(5)<=EPS; N(6)<=EPS; N(7)<=EPS; !EPS<=0; !S(5)=0;

X(1)<=Y(1,1)+Y(1,4)+Y(1,5)+Y(1,7);
X(2)<=Y(2,2)+Y(2,4)+Y(2,6)+Y(2,7);
X(3)<=Y(3,3)+Y(3,5)+Y(3,6)+Y(3,7);

!POOL CONSTRAINTS;
Y(1,1)<=V(1)*S(1); Y(2,2)<=V(2)*S(2);
Y(3,3)<=V(3)*S(3); Y(1,4)+Y(2,4)<=V(4)*S(4);
Y(1,5)+Y(3,5)<=V(5)*S(5); Y(2,6)+Y(3,6)<=V(6)*S(6);
Y(1,7)+Y(2,7)+Y(3,7)<=V(7)*S(7);

!POOL SELECTION;
S(1)+S(4)+S(5)+S(7)=1; !PLAYER 1;
S(2)+S(4)+S(6)+S(7)=1; !PLAYER 2;
S(3)+S(5)+S(6)+S(7)=1; !PLAYER
3;

!PROFITGOAL;
X(1)+NP(1)-PP(1)=TARGET(1,1);
X(2)+NP(2)-PP(2)=TARGET(2,1);
X(3)+NP(3)-PP(3)=TARGET(3,1);

!FAIRNESSGOAL;
!PLAYER 1;
Y(1,4)-Y(2,4)+NF(1,4)-PF(1,4)=TARGET(1,2)*S(4);
```

```
Y(1,5)-Y(3,5)+NF(1,5)-PF(1,5)=TARGET(1,2)*S(5);
Y(1,7)-(Y(2,7)+Y(3,7))+NF(1,7)-PF(1,7)=TARGET(1,2)*S(7);

!PLAYER 2;
Y(2,4)-Y(1,4)+NF(2,4)-PF(2,4)=TARGET(2,2)*S(4);
Y(2,6)-Y(3,6)+NF(2,6)-PF(2,6)=TARGET(2,2)*S(6);
Y(2,7)-(Y(1,7)+Y(3,7))+NF(2,7)-PF(2,7)=TARGET(2,2)*S(7);

!PLAYER3;
Y(3,5)-Y(1,5)+NF(3,5)-PF(3,5)=TARGET(3,2)*S(5);
Y(3,6)-Y(2,6)+NF(3,6)-PF(3,6)=TARGET(3,2)*S(6);
Y(3,7)-(Y(1,7)+Y(2,7))+NF(3,7)-PF(3,7)=TARGET(3,2)*S(7);

!POOL SELECTION VARIABLES ARE BINARY; @FOR(POOL:@BIN(S));
END
```